WHAT DYNAMIC CHANGES IN THE SUN DRIVE THE EVOLUTION OF OSCILLATION FREQUENCIES THROUGH THE ACTIVITY CYCLE?

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ABSTRACT

The frequencies of solar oscillations (f- and p-modes) evolve through the solar cycle. The changes are driven by some combination of changes in the magnetic field, thermal structure and velocity field. It is unclear what is the precise combination of the three. One way or another, this thorny issue rests on an understanding of the response of the solar structure to increased magnetic field, but this is complicated. As well, we do not understand the origin of the sun's irradiance increase with increasing magnetic activity.

Until recently, it seemed that an unphysically large magnetic field change was required to account for the frequency evolution during the cycle. However, the problem seems to have been solved (Dziembowski, Goode & Schou 2001). Specifically, a small-scale magnetic field was considered assuming uncorrelated field components – allowing the vertical component to be statistically different from the two horizontal ones. It turns out that a purely radial random field is the most economical, as well as being more physically sensible for other reasons. Furthermore, the solution might have a direct bearing on the origin of the irradiance variation. We discuss recent results and the present state of our knowledge.

Key words: Helioseismology; Sun's Radius; Solar Irradiance.

1. INTRODUCTION

The physics of solar irradiance change over the activity cycle is not well understood, and is the subject of controversy. The competing models are ones in which the sun is hotter at higher activity (e.g., Kuhn 2000), and ones in which the sun is cooler at higher activity (e.g., Spruit 2000). In the latter picture, higher irradiance is explained by a corrugated surface rendering the sun a more effective radiator. So, is the active sun hotter or cooler than the inactive sun?

Naively, the issue would seem to be clear. That is, considering the sun to be a blackbody, we have

$$\frac{\Delta_{\min}L}{L} = \frac{4\Delta_{\min}T}{T} + \frac{2\Delta_{\min}R}{R}, \qquad (1)$$

where the change, $\Delta_{\min}$, is with respect to activity minimum. Further, assuming that the irradiance varies like the luminosity and that $\frac{\Delta_{\min}R}{R}$ is negligible, one would conclude that the sun is hotter at activity maximum since the irradiance is greater there – by 0.1%. However, the truth is more subtle, and in our naive assumptions, $\frac{\Delta_{\min}T}{T}$ is actually a proxy for some combination of the evolving magnetic field, thermal structure and turbulent pressure. Thus, we have naively cast these three candidates as a temperature increase in the simple blackbody equation, while it could well be that the sun is actually cooler at activity maximum. It turns out that the cycle dependent radius changes (Brown&Christensen-Dalsgaard 1998, Emilio et al. 2000, and Dziembowski, Goode & Schou 2001) are too small to matter for irradiance changes.

Spherically symmetric changes in the sun are manifest in the shifts of centroid frequencies in the spectrum of solar oscillations through the solar cycle. Such shifts were first reported by Woodard and Noyes (1985) during the declining phase of cycle 21. This result has been confirmed and refined by many subsequent investigations. One very recent inference from the frequency changes of the solar f-modes was that the sun actually shrinks with increasing activity, Dziembowski, Goode & Schou (2001).

The evolution of oscillation frequencies provides the most accurate measures of cycle dependent changes in the sun. The real challenge that remains is a precise connection between these global, seismic measures and characteristics of the dynamic sun. There are discrepant views as to the connection.

One way or another all of these thorny issues rest on an understanding of the response of the solar structure to an increasing magnetic field with increasing activity, but this is complicated. Goldreich et al. (1991) specifically proposed that changes in the superficial, random magnetic field is the primary cause of the centroid frequency shifts. This idea has been criticized by Kuhn (1998) who points out that Goldreich et al. require an r.m.s., quadratic, near-surface magnetic perturbation, $< B^2 >$ of around $(250G)^2$, while the observations of Lin (1995) and Lin & Rimmele (1999) show an increase of the mean surface field which is significantly weaker ($< B^2 >\sim (70G)^2$). Instead, Kuhn sees a critical role for the variations of the Reynold's stresses or turbulent pressure through the solar cycle. He also proposes that changes in the aspherical component of the stresses are responsible for the varying symmetric part of the spectrum of solar oscillations (the so-called even-a coefficients). Clearly, we are lacking a basic understanding of how the frequency changes arise, and so, we also do not understand the origin of the aforementioned dynamical changes in the sun through the activity cycle. However, Dziembowski, Goode & Schou (2001) used SOHO/MDI seismic data to shed light on the character of the dynamical changes with rising activity by using f-mode (f-modes are the eigenmodes of the sun having no radial null points and these modes are asymptotically surface waves) and p-mode data to probe the evolution of the size of the sun as activity increases. This knowledge can be used to guide us about the nature of the dynamic changes giving rise to "$\frac{\Delta_{\min} T}{T}$" in the blackbody equation.

2. THE ORIGIN OF THE CHANGING HELIOSEISMIC RADIUS OF THE SUN

All helioseismic determinations of the solar radius to date have relied on the following asymptotic relation for f-modes frequencies,

$$\frac{\Delta\nu_\ell}{\nu_\ell} = -\frac{3}{2}\frac{\Delta R}{R}, \tag{2}$$

where the Δ now implies a difference between true and model values. With this, Schou et al. (1997) derived a helioseismic radius of the sun that is quite close to the photospheric radius deduced by Brown & Christensen-Dalsgaard (1998)from several years of transit observations. The seismic determination rests on the radius of the f-modes scaling with the sun's true radius, which allows us to compare true and model photospheric radii. These the seismic and transit photopheric values are 300-400 km smaller than the radius that has been used in standard models of the sun. Applying Eq.(2) to determine radius changes through the solar cycle is fraught with difficulties.

Antia et al. (2000) pointed out that using this relation for modes with ℓ extending up to 300, as Dziembowski et al. (1998) did, is not justified because of significant departures from $\nu \propto R^{-1.5}$ are present in higher ℓ's that can change the seismic radius at the level of their quoted errors. The departure increases with ℓ, which as Brown (1984) first suggested could be accounted for as an effect of turbulence in the upper convective zone. However, surface magnetic fields may also have significant effects on f-mode frequencies (Evans & Roberts, 1990; Jain & Roberts). With these two sources of perturbation to f-mode frequencies, we must contemplate solar cycle changes beyond that of a simple radius change. The relative contribution of the near-surface changes are expected to increase with ℓ, because such changes should be inversely proportional to mode inertia, I_ℓ, which sharply decreases with ℓ.

There is another problem in applying Eq.(2) in a search for the radius variations correlated with activity. This problem follows from the fact that the induced modifications are quite non-uniform, and each f-mode has it is own radius, R_ℓ, which is given by

$$R_\ell = \left(\frac{1}{I_\ell}\int r^{-3} dI_\ell\right)^{-1/3}. \tag{3}$$

For high degree modes, the f-mode radii are close to the solar radius. The values of R_ℓ/R range from 0.9883 at $\ell = 100$ to 0.9946 at $\ell = 300$. While we have $R_\ell \approx R$, a corresponding approximation for cycle dependent changes $\Delta_{\min} R_\ell$ is quite problematic. When the f-mode frequencies were used to refine the value of the radius for modeling the sun, we could expect an approximate, homologous relation, $R_\ell \propto R$. But such a relation cannot be expected in the case of the activity induced changes, which are quite small and seem to be confined to the outermost part of the sun. Then, the inferred value of ΔR in Eq.(2) would refer to the range of depths beneath the photosphere corresponding to the range of ℓ's in the data sets. Antia et al. (2000) used modes in the 100–200 range, which translates to 10–6 Mm in depth. Their finding implies that this layer was moved downward by about 5 km during the two years they considered. The truth is, with these data, we cannot say anything about what happened in the layers above. Thus, we have no information about the evolution of the photospheric radius of the sun. However, we can overcome this problem.

2.1. Formal determination of the rate of shrinking from f-modes

To account for the effect of the near-surface changes on f-mode frequencies and possible differential changes, we use the formulation of Dziembowski, Goode & Schou (2001), who showed the benefit of modifying Eq.(2) into

$$\Delta_{\min}\nu_\ell = -\frac{3}{2}\frac{\Delta_{\min} R_f}{R}\nu_\ell + \frac{\Delta_{\min}\gamma_f}{I_\ell}, \tag{4}$$

where ΔR_f denotes the radius change inferred from a particular set of f-modes. In this formulation, the parts of the frequency change due to the size change and the near surface effects cleanly separate and removes the ℓ-dependent anomaly that would occur if we were to use Eq.(2), see Fig.1. However, the size

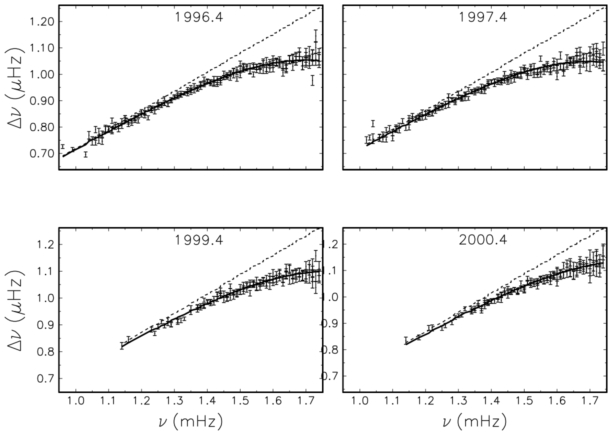

Figure 1. Differences between measured and calculated f-mode frequencies. The error bars show estimated standard deviations of measured values. The dates correspond to the center of the individual 72-day long measurement periods. The solar model was calculated assuming $R_\odot = 695.991$ Mm. The solid line represents the fit to Eq. (4). The dashed straight line represents the part attributed to the difference between the solar radius and that adopted in the model.

change refers to a region 5-10 Mm beneath the solar surface, corresponding to the range of radii, R_l, from Eq. (3) for the MDI f-mode data. Thus, the evolving frequencies contain information about a band 5-10 Mm beneath the solar surface –the f-mode radius band, or heretoforward the "f-mode radius", which is not to be confused with the seismic radius. However, the evolution of the f-mode frequencies can be used to reveal whether the sun's f-mode radius shrinks or expands. To determine the photospheric radius change from the seismic data, we must use the near surface data to tell us whether the last few megameters of the sun contracts or expands with increasing activity. The change of the photospheric radius is the sum of the two.

In Fig. 1, we see the departure from the linear relationship implied by the radius adjustment sharply increases with ν. Antia et al. (2000) considered only modes with $\nu < 1.44$ mHz, and it seems that the departure from a straight line is still small. However, this is somewhat misleading because we used a model with much too large a radius. As we see in Table 1, at the level of changes of a few nanohertz (i.e. radius changes of a few km), the difference is quite significant. We emphasize that high ℓ-modes are important because with increasing ℓ, R_ℓ approaches the solar radius. For such modes, including γ_f is essential, which implies that we have to rely on Eq.(4) rather than Eq.(2). With Eq.(2), we get a much poorer fit ($\chi^2 = 4.4 - 16.5$) and the correction to the solar radius is larger by some 20 km. This illustrates the trade-off – increasing ℓ moves us closer to the surface, but such high ℓ's are more strongly contaminated.

In Fig. 2, we show the variations of the f-mode radius and γ_f inferred from f-modes from the truncated

Table 1. Contributions to f-mode frequency shifts during the rising phase of cycle 23

ℓ	ν_ℓ[mHz]	I_ℓ	$\Delta\nu_R$[μHz]	$\Delta\nu_\gamma$[μHz]
100	1.02	381	0.010	0.0012
130	1.15	165	0.011	0.003
200	1.43	39.	0.014	0.012
300	1.74	9.4	0.017	0.050

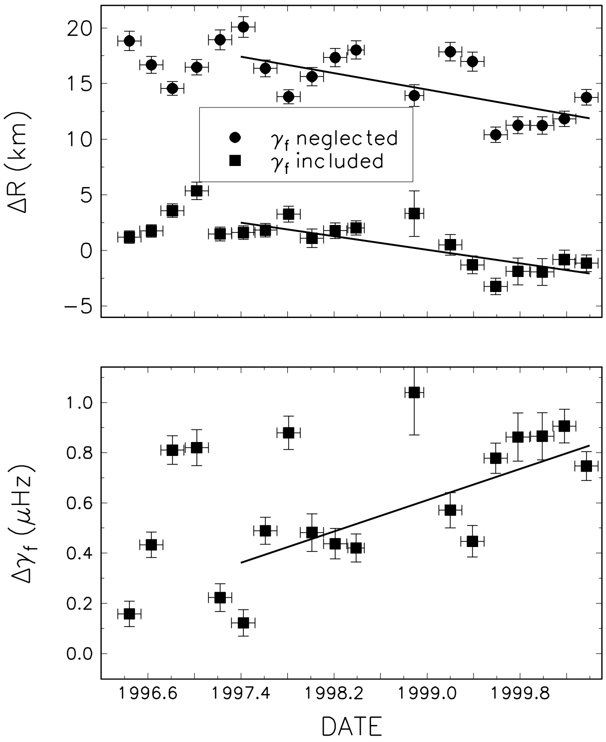

Figure 2. Upper panel: Variation of solar radius between 1996.4 and 2000.4 inferred from f-mode frequencies with and without the γ_f-term. Two straight lines represent linear fits to the data starting from 1997.4 when the rise of cycle 23 began. Lower panel: Corresponding variation of γ_f, which describes remaining near-surface contribution to f-mode frequency variations.

data sets. The rise of the current activity cycle began in 1997.4 which was marked by a sharp rise of the seismic activity indicators (Dziembowski et al. 1998, or the lower panel of Fig. 6 here). There is a corresponding sharp rise of p-mode frequencies beginning at this time. That is why we choose 1997.4 to begin our linear fits in Fig. 2. We have no explanation, as yet, for the relatively large fluctuations in ΔR_f which appear to have a one-year period. For comparison, we also show the result obtained when the γ_f-term is ignored. There is a difference, but not as large as one might anticipate by looking at Fig.1. The rate of radius decrease is only insignificantly higher than in our standard version, and the error is larger.

In detail, we found from our linear fit, with the γ_f,

$$\frac{dR_f}{dt} = (-1.51 \pm 0.31) \quad \text{km/y}, \tag{5}$$

and without the γ_f-term,

$$\frac{dR_f}{dt} = (-1.82 \pm 0.64) \quad \text{km/y}.$$

The values are similar to those found by Antia et al. (2000). To make a closer comparison, we truncated our data sets at $\ell = 200$, and then we found

$$\frac{dR_f}{dt} = (-1.80 \pm 0.38) \quad \text{km/y}.$$

Having in mind that we still miss modes between $\ell = 100$ and 137, it is fair to say that there is no disagreement between our findings and theirs, implying that at a depth of from 6 to 10 Mm the sun shrank by some 4 to 6 km during the rising phase of this activity cycle.

How reliable is this finding? The main concern is the role of the near-surface perturbation and the cross-talk between the two terms on the right hand side of Eq.(4). In the lower panel of Fig. 2, we show the γ's. The linear fit for γ, which is visibly poorer, but not too bad, yields

$$\frac{d\gamma_f}{dt} = (0.180 \pm 0.051) \quad \mu\text{Hz/y}. \tag{6}$$

The relative contribution of the two terms to overall f-mode frequency variations depends on ℓ. Again, in Table 1, we compare these two contributions, denoted by$\Delta\nu_R$ and $\Delta\nu_\gamma$ for selected ℓ-values. The increasing role of $\Delta\nu_\gamma$ is a consequence of decreasing mode inertia. It should be noted that $\Delta\nu_\gamma$ yields an appreciable contribution to $\Delta\nu$ even for modes with $\ell \leq 200$. Caution is necessary, but we will proceed further assuming that the effect is indeed real.

Even as small as it seems, a shrinking of the sun's radius during the rising phase of activity is not easy to explain. To investigate, we write the Lagrangian change of the local radius in the form

$$\Delta r(r_0) = r - r_0 = -\int_{r_b}^{r_0} \frac{\Delta\rho}{\rho} \left(\frac{x}{r_0}\right)^2 dx, \tag{7}$$

where r_b is the radius at the bottom of the layer perturbed by activity, and r_0 is the radius at a specified fractional mass, M_r/M, at activity minimum and $\Delta\rho$ denotes the horizontally averaged change of density. We obtain a more revealing form of Eq.(7) by expressing $\Delta\rho$ in terms of the averaged entropy and magnetic field changes.

For the horizontally averaged gas pressure in the presence of a random, r.m.s. magnetic field we have, after Goldreich et al. (1991), but generalized it so that the random field is not necessarily isotropic,

$$\Delta P_g = -\Delta(\beta P_m), \tag{8}$$

where

$$P_m = \frac{\overline{B_h^2} + \overline{B_r^2}}{8\pi}$$

is magnetic pressure and

$$\beta = \frac{\overline{B_h^2} - \overline{B_r^2}}{8\pi P_m}$$

is a measure of the statistical anisotropy of the r.m.s. field.

With the use of thermodynamical relations, we determine

$$\begin{aligned} \Delta r &= \int_{r_b}^{r_0} \left[\frac{1}{\Gamma_1}\frac{\Delta(\beta P_m)}{P_g}\right. \\ &+ \left.(-\rho_T)\frac{\Delta S}{c_p}\right] \left(\frac{x}{r_0}\right)^2 dx, \end{aligned} \tag{9}$$

where ρ_T denotes the logarithmic derivative of density at constant pressure. The remaining thermodynamical quantities have their standard meanings. At the relevant depths, the gas is nearly ideal. Thus, we may use $\rho_T = -1$, $1/\Gamma_1 = 0.6$, and find

$$\frac{\Delta S}{c_p} = \frac{\Delta T}{T} - 0.4\frac{\Delta P_g}{P_g}.$$

If we attribute all of the shrinking to thermodynamic effects, how big can it be? The irradiance from an active sun is higher than average. If the same is true about luminosity then we should have $\Delta S < 0$. Hence, a negative contribution to Δr. However, this must be very small. Roughly, the increase in luminosity from activity minimum to maximum is given by the therodynamic relation between heat loss and entropy decrease ($dQ = TdS$)

$$\Delta L \sim \frac{\Delta S}{\Delta t_{\rm cyc}} \int_{\rm CZ} T dM_r,$$

where the integral yields the mean temperature of the convection zone, ΔS is entropy change, $\Delta t_{\rm cyc}$ is the length of the solar cycle and ΔL is the luminosity change. The thermal timescale of the convection zone, the time for the energy stored in the convection zone to be released by the luminosity is given by

$$\Delta t_{\rm CZ} \sim \int_{\rm CZ} c_p T dM_r / L.$$

Combining the two, we have

$$\frac{\Delta S}{c_p} \sim \frac{\Delta L}{L}\frac{\Delta t_{\rm cyc}}{\Delta t_{\rm CZ}} \sim 10^{-3}\frac{10}{10^5} = 10^{-7}.$$

This corresponds to a radius change of order 0.1 km, or so, over the rising phase of the cycle, which is an order of magnitude, or so, smaller than the result we have just seen. A more acceptable explanation for the f-mode radius change would be a variation in the magnetic field. The consequences of a magnetic field increase depend on β. For a purely radial field ($\beta = -1$), the increase implies contraction – as deduced from the f-mode data. For an isotropic field

($\beta = 1/3$) the increase implies expansion – contrary to what is deduced from the f-modes. Thus, we have a non-trivial constraint on the change of the internal magnetic field, and the field geometry implying the minimum increase to account for the rate of the shrinking corresponds to $\beta = -1$. Then, we have $\Delta < B >_{\rm rms} = (\overline{\Delta(B_r^2)})^{1/2}$. Again, an isotropic random field would seem to be precluded for the region of the f-mode radius, and the region immediately beneath because it implies an expansion, rather than a contraction.

It must be emphasized that any inference regarding the change of the solar radius itself is limited by the lack of accurate information about what happened in the outer 4 Mm of the solar interior. This is the region where we may expect the largest activity induced variations for two reasons. First, the rapid decline of gas pressure and second, the thermal structure of this layer is more susceptible to changes in the efficiency of the convective energy transport induced by the field changes. The f-mode data we have at hand provide some information about changes in this layer through the γ_f. Similar, but much more accurate information is available in the p-mode data, which we now consider.

The p-mode spectrum of MDI frequency data is about 13 times richer than that for f-modes. Unfortunately, p-modes are not directly useful for determining changes in the solar radius. The simple relation, $\nu \propto R^{-1.5}$ would be valid for p-modes only if the changes were homologous throughout the whole sun. This is far from the truth for the changes we are considering here. However, from p-modes one may make a much more precise determination, than from f-modes, of the near-surface perturbation. For p-modes, we call it $\Delta_{\rm min}\gamma_p$, and it describes frequency changes caused by a variable perturbation localized near the surface. Here $\Delta_{\rm min}$ is with respect to the activity minimum (1996.4 in the SOHO/MDI data set). For the spherically symmetric part of the p-mode γ's, we have, following Dziembowski, Goode & Schou (2001), we have

$$\frac{d\gamma_{p,0}}{dt} = (0.149 \pm 0.008) \quad \mu{\rm Hz/y}. \tag{10}$$

The dependence of $\gamma(\nu)$ yields an important constraint on the localization of the source of solar cycle variations in p-mode frequencies.

Following our generalized form of Goldreich et al. (1991), we link the frequency change to the change of the mean squared magnetic field and a Lagrangian change of a single thermodynamic parameter. For the latter, we prefer to use temperature rather than entropy which was used by Goldreich et al. From Eqs. (14) and (15) of Goldreich et al., we get the following expression for the change of γ_p,

$$\begin{aligned}\Delta\gamma_p &= \frac{1}{8\pi^2\nu}\int d^3\vec{x}|{\rm div}\vec{\xi}|^2\{P\Gamma_1(1+\Gamma_\rho)\rho_T\frac{\Delta T}{T}\\ &+ [1-\beta\Gamma_1(\Gamma_P+\Gamma_\rho\rho_P)\\ &- \beta(\Gamma_1 - 1 + \Gamma_1\rho_P)]\Delta P_m\}. \end{aligned}\tag{11}$$

Here, we denote by Γ_P and Γ_ρ, the logarithmic derivatives of Γ_1. The ideal gas equation cannot be used in the layers where most of the contribution to $\Delta\gamma_p$ arises.

Goldreich et al. (1991) explained the p-mode frequency changes during the rising phase of cycle 22 in terms of magnetic field and temperature changes, with former being dominant and causing the frequency increase. They invoked a chromospheric temperature increase to explain the reversal in the increasing trend in $\Delta(\nu)$ in the BBSO data. We do not see such a trend in the SOHO/MDI data. Thus, as a first guess we interpret $\Delta\gamma_p$ in terms of magnetic field changes. Later, we will discuss other sources of the p-mode frequency changes.

In our generalized formulation of Goldreich et al., we considered two values of β, -1 and 1/3, and the following form for the depth, D, dependence of magnetic field increase

$$\Delta < B >_{\rm rms} = \begin{cases} B_b & \text{if } D \geq D_b \\ B_b + \lambda\left(\frac{D-D_b}{D_b-D_m}\right) & \text{if } D_t < D < D_b \\ \Delta < B >_{\rm rms}(D_t) & \text{if } D \leq D_t \end{cases}$$

where $D_m = -0.49$Mm denotes D at the temperature minimum, and B_b, D_b, and λ were determined by fitting the three terms in the series given just above. For D_t we adopted either D_m or 0.

In Fig. 3 we show two examples of the field's changing behavior that would be consistent with the observed γ's, and compare them with two cases that are clearly inconsistent. One of the two inconsistent cases is a depth independent increase, and the other is an example of the field gradually increasing to about 3 kG at 8Mm. In all four examples, we used $\beta = -1$. We see that indeed the $\gamma_p(\nu)$ provides a strong constraint on the localization of the source of frequency changes, but clearly not a unique answer. For the two fitted cases, the inferred values of B_b are 290 and 250 G. Corresponding values of $B_{\rm ph} \equiv \Delta < B >_{\rm rms}(0)$ are 62 and 94 G. An equally good fit was obtained with the choice $\beta = 1/3$. Data on the three models of the magnetic field change fitting $\Delta\gamma_p(\nu)$ data are given in Table 2. The result for $\beta = 1/3$ is not significantly different from that found by Goldreich et al. (1991). For the latter case, to explain the p-mode frequency increase between minimum and maximum, we require an increase of the rms magnetic field growing from 200 G in the photosphere to 840 G at 4.25 Mm. The corresponding numbers of Goldreich et al. are 250 and 1000 G.

For $\beta = -1$, the required field is significantly smaller than for β=1/3, and it is arguably consistent with the observed photospheric values for the r.m.s. field growth observed by Lin and Rimmele (1999). Certainly, β=-1 is the most economical form of the field to account for the cycle-dependent frequency changes, but does either argument imply that we can exclude an isotropic random field in the outermost layers? Well, f-mode radius shrinking with increasing activity argues for β=-1, and it is difficult to imagine how the buoyant field, which is observed to radial at the surface and constrained to be radial beneath 5 Mm below the surface, would become isotropic in the 5 Mm band in between. Further, the p-modes contain not only the spherically symmetric part of the near surface term, γ_0, but also higher order terms, γ_1(P_2-distortion), γ_2(P_4-distortion), etc., which are plotted in Fig. 4. We note that the low-

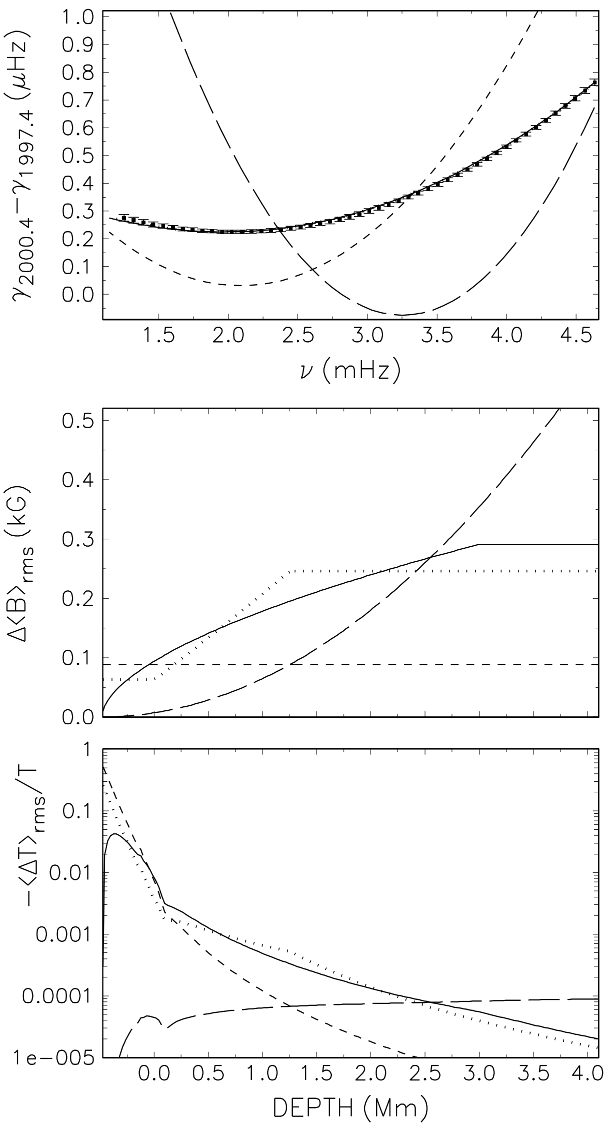

Figure 3. In the top panel, points with the error bars represent $\Delta\gamma_0(\nu)$ inferred from p-mode frequency difference between 2000.4 and 1997.4. The lines correspond to various distributions of the averaged magnetic field, shown in the middle panel. The solid and dotted lines are within the error bars in the top panel. The bottom panel shows the relative temperature decreases required to cause similar frequency shifts as the corresponding magnetic field increases.

order, asymmetric γ's are generally much larger than γ_0. This is true for β=-1, but not for β=1/3, Dziembowski and Goode (2002). Thus, the problem of a too large photopsheric field growth seems to be resolved. But what about the role of the changes in thermal structure and turbulent pressure with rising activity?

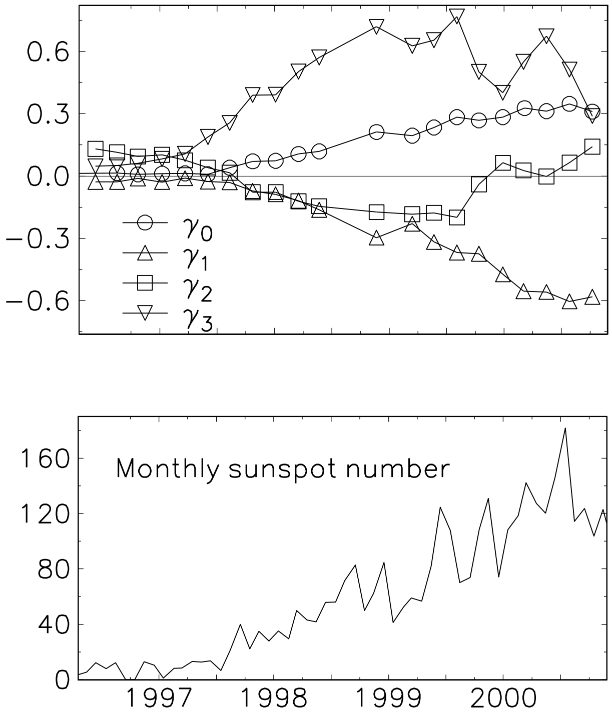

Figure 4. In the top panel, the behavior of γ_0 is shown as a function of time from the SOHO/MDI data, γ_0 is defined with respect to the 1996 activity minimum. The lower panel shows the corresponding sunspot number, which tracks γ_0 closely. Also, shown in the upper panel is the evolution of γ_1 through γ_3 – the P_2, P_4 and P_6 shape asymmetries, which are generally much larger than that for P_0.

In the bottom panel of Fig. 3, we plot the relative temperature changes, which gives the same local contributions to $\Delta\gamma_p$ as the corresponding changes in the magnetic field. We see that the required change of temperature is unacceptably large in the atmospheric layers. However, in sub-photospheric layers we cannot exclude the rms $\Delta T/T$ at the 10^{-3} level. Such a temperature decrease would be a significant contributor to the observed frequency increase. Brüggen and Spruit (2000) argue that one expects a lower subsurface temperature from an increasing magnetic field, and that the effect should be searched for by means of helioseismology. A contribution from temperature decrease would lower the requirement for the magnetic field increase in the sub-photospheric layers.

Yet another potential contributor to the frequency increase is a decrease in the turbulent velocity. Roughly, the relative change in the turbulent velocity, $\Delta v_t/v_t = q$, has the same effect as a relative temperature change $\Delta T/T = 0.5q\mathcal{M}^2$, where $\mathcal{M}$ is the turbulent Mach number. In the sub-photospheric layers, $\mathcal{M}$ is in the 0.1–1 range. Thus, the effect may be significant, and we may expect a decrease in v_t, with increasing activity, because the magnetic field should inhibit convection, which in turn yields shrinkage because decreasing the turbulent pressure suppresses its forcing of a larger radius.

Thus, the combined effects of the thermal and turbulent pressure changes with increasing activity can account for part of the perturbation, which serves to reduce the required field growth.

In Table 2, we provide the values of the contribution to the rate of the photospheric radius change due to the magnetic field increase inferred from the γ_p changes. We emphasize that the rate does not refer to photosphere but to the mass point corresponding to the unperturbed (solar minimum) photosphere and that the value does not include the part that was inferred from f-mode frequency changes.

Combining the shrinkage of the f-mode radius with the implied contraction of the outer few megameters, we are left with an implied photospheric radius shrinkage of 2-3 km/year with rising activity. We do not view this as being fundamentally inconsistent with the growth rate of about 5.9±0.7 km/y determined by Emilio et al. (2000) from the direct radius measurements based on SOHO/MDI intensity data. Perhaps the difference may be explained by the difference between their measurements of limb intensity, dR_d/dt, and our $(dR/dt)_{\rm ph}$. Both results, however, imply a negligible contribution of the radius change to the solar irradiance variations. Furthermore, the two estimates of the radius change between maximum and minimum activity are by two orders of magnitude less than found by Nöel (1997) from his measurements with the astrolabe of Santiago. He finds the difference between the 1991 (previous maximum) and 1996 radii which is exceeding 700 km. The data from the Solar Diameter Monitor (Brown & Christensen-Dalsgaard, 1998) are inconsistent with such large variations, although there is a hint of possible radius increase during 1987 of some 30–40 km. On the other hand, a theoretical constraint on radius given by Spruit (1994) is even tighter than than that from helioseismology. The number he quotes for the maximum to minimum difference is $2 \times 10^{-7} R_\odot = 0.14$ km.

Based on our result of a more physically reasonable magnetic field describing the frequency changes, we were led to the probe the question of whether the sun is hotter at activity maximum.

3. IS THE SUN HOTTER AT ACTIVITY MAXIMUM?

Dziembowski and Goode (2002) have developed a new formalism for describing the frequency changes in the spectrum of solar oscillations, and here we initially exploit that formalism with a very simple model. The starting point of our new development is the variational principle for stellar oscillations in which we treat the effect of stellar magnetic fields as a small perturbation. We have developed a similar formulation that includes various forms of the velocity field. However from our preliminary analysis, Dziembowski and Goode (2002), we have shown

Table 2. Inference from p-mode frequency changes between 1997.4 and 2000.4

β	λ	D_b	D_t	$B_b[G]$	$B_{\rm ph}[G]$	$(dR/dt)_{\rm ph}$
-1	0.58	3.00	-0.49	29	62	-1.3 km/y
-1	1.15	1.27	0.00	25	94	-1.1 km/y
1/3	0.62	4.25	-0.49	84	200	2.3 km/y

that the role of global flow changes, including rotation and meridional circulation, are far too small to play a role. Further, acceptable changes of the turbulent velocity also seem to be too small to be significant. Specifically, if we attempt to attribute the observed modal frequency variations to random velocity changes, then the required changes seem to be too large. That is, about 300 km of the radius is due to the effect of turbulent pressure, Nördlund (2002). The radius seems to change by about 1% of that value in going from activity minimum to maximum, but changes in the turbulent velocity, roughly on the order of 10% seem to be required to account for the frequency shifts. We are checking this inconsistency in detail, but it seems that the required velocity changes are too large to be physically reasonable. Thus, we are left with the magnetic field and temperature. The two are not independent. However, for the spherically symmetric part, the link goes though the condition of thermal equilibrium, which is difficult to treat. For the aspherical part, the link is simple and follows from the condition of mechanical equilibrium. Thus, we can express the even-a coefficients in terms of the magnetic field alone.

Specifically, we consider a small-scale magnetic field assuming uncorrelated field components, but allowing the vertical component (depending on r) to be statistically different from the two horizontal (depending on θ and ϕ) ones. This formulation is the same as our earlier generalization of that of Goldreich et al. (1991), except that we further generalize to take into account the angular dependence of the field so as to analyze the the fine structure in the spectrum of solar oscillations beyond that due to rotation (i.e., analyze the so-called even-a coefficents). Our averaged values, $\overline{B_j^2}$, are thus treated as general functions of r and slowly varying functions of the co-latitude. The latter dependence is represented in the form of a truncated Legendre polynomial series,

$$\begin{aligned}\overline{B_jB_k} &= \delta_{jk}\sum_{k=0}\Big[\delta_{rk}\mathcal{M}_{r,k}(r) \\ &+ \frac{1}{2}\mathcal{M}_{H,k}(\delta_{\theta k}+\delta_{\phi k}\Big]P_{2k}(\cos\theta), \quad (12)\end{aligned}$$

where we include only seismically detectable (the ones that are symmetric about the equator) terms.

Each of the k-components gives rise to a P_{2k} distortion of the sun's structure. For $k > 0$, the hydrostatic equilibrium condition suffices to determine the distortion of all thermodynamical parameters.

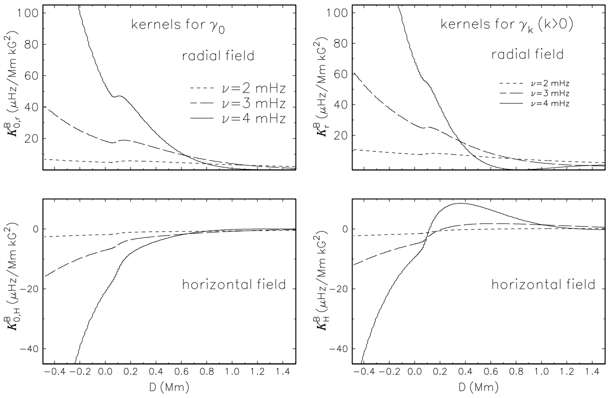

Figure 5. Kernels for the γ's at three selected frequencies plotted as functions of the depth in outer part of the standard solar models.

In particular, for the $k > 0$ component of the relative Lagrangian perturbation of the temperature, we have

$$\left(\frac{\Delta T}{T}\right)_k = \frac{1}{8\pi\chi_T}\left\{\mathcal{M}_{r,k}\left[1+2\chi_\rho\left(\frac{d\ln\mathcal{M}_{r,k}}{d\ln r}\right.\right.\right. - \left.\left.\left(\frac{d\ln\rho}{d\ln p}\right)\right] - \mathcal{M}_{H,k}\left[1 + \chi_\rho\left(\frac{d\ln\mathcal{M}_{H,k}}{d\ln r}\right) - \left(\frac{d\ln\rho}{d\ln p}\right)\right]\right\}, \tag{13}$$

where the Δ is with respect to the spherical sun. We emphasize here that for $k = 0$, the meaning of Δ is different. It is usually taken to be with respect to activity minimum. Here we adopted the standard notation with χ_T and χ_ρ denoting the logarithmic derivatives of pressure with respect to temperature at constant density and that with respect to density at constant temperature, respectively.

If the magnetic perturbation is significant only in the layers well above the lower turning points for all p-modes considered, then all the frequency changes, $\Delta\nu$ and the even a coefficients, may be expressed through the γ's using kernels that are ℓ-independent. Owing to a different constraint from hydrostatic equilibrium, we have different expressions for γ_k, depending on whether $k = 0$ or $k > 0$. In the former case, the temperature perturbation is treated as an independent parameter. In that case, we have

$$\gamma_0 = \int\left(\mathcal{K}_{0,T}\frac{\Delta T}{T} + \mathcal{K}^B_{0,r}\Delta\mathcal{M}_{r,0} + \mathcal{K}^B_{0,H}\Delta\mathcal{M}_{H,0}\right)dD, \tag{14}$$

where D is depth. All the kernels, $\mathcal{K}$, may be expressed in terms of parameters of the standard solar model and the radial eigenfunctions of its p-modes.

Goldreich et al.(1991) considered only changes in centroid frequencies. They pointed out that to explain the frequency increase between 1986 and 1988, a 1% decrease of the photospheric temperature is needed ($\mathcal{K}_{k,T}$ is always < 0). They regarded this requirement as being incompatible with observations, and adopted the changing magnetic field as the sole cause of the frequency increase; they found that the field increase must be about 250 G at the photosphere, and steadily growing to about 1 kG at a depth of 10 Mm. Their numbers refer to the case of a statistically isotropic field ($\mathcal{M}_{H,k} = 2\mathcal{M}_{r,k}$). A much more modest field increase (< 100 G at the photosphere) would result for an inwardly growing, purely radial, random r.m.s. field (Dziembowski, Goode & Schou, 2001).

For $k > 0$, the temperature perturbation may be eliminated, and we have a simpler expression,

$$\gamma_k = \int(\mathcal{K}^B_r\mathcal{M}_{r,0} + \mathcal{K}^B_H\mathcal{M}_{H,k})dD. \tag{15}$$

For Eq.(15), note that the kernels are the same for all k's. This follows both from $k \leq \ell$ and our assumption that the perturbation is significant only above the inner turning points of the oscillations.

In Fig.5, we show these kernels in the outer layers of the sun. We note that the strong sensitivity to the frequencies emphasizes the probing power of the $\gamma(\nu)$. We remark that the relative success of fitting frequency differences with a constant γ reveals that the dominant perturbation must be located in a region in which all the kernels are not too different. That is, just beneath the photosphere. Further, a critical point is that by inspecting the kernels, one can easily see that they imply that *a radial field increase in the outer layers will lead to a frequency increase, while an increase of the horizontal field is very likely to have an opposite effect.* That is, since oscillation frequencies increase with increasing activity, the behavior of the kernels tells us the most economical form of the field to describe the frequency changes. In the outer layers, the signs of the kernels are $\mathcal{K}^B_{k,r} > 0$ and $\mathcal{K}^B_{k,H} < 0$. Thus, the most economical requirement is a purely radial field. The novel aspect of our present formulation is that instead of searching for the magnetic field through frequency dependent γ coefficients, we now express γ directly through the magnetic field parameter that we propose to infer.

As an illustration of this new approach to show the probing power of the γ's, we consider a simple functional form of the depth dependence of the radial field,

$$\Delta < B >^2_{\rm rms} = \begin{cases} B_b^2 & \text{if } y > 1 \\ B_b^2 y^2(3-2y) & \text{if } y \leq 1 \end{cases}, \tag{16}$$

where

$$y = \frac{D_{\min} - D}{D_{\min} - D_b},$$

and where $D_{\min}$ is the depth at the temperature minimum, and D_b is the adjustable parameter. In Fig.6, we show behavior of χ^2 with varying D_b. In the left panel we show the value of χ^2 for k=0 and 1

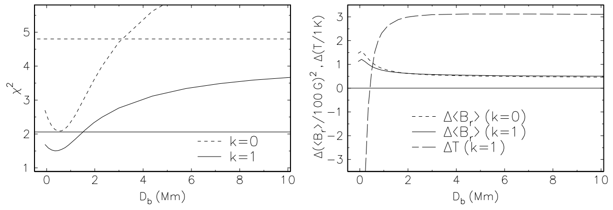

Figure 6. Left panel shows the χ^2 from fitting γ_0 and γ_1 to Eqs.(8) and (9) with B_r given by Eq.(10) with D_b treated as the adjustable parameter. Flat dashed and solid lines are for constant γ's. Right panel shows the implied field intensity and the temperature change.

for a range of choices of D_b. The two minima are at about the same value of D_b with the k=0 term showing a more robust minimum. In the right panel we show the implied radial field intensity as a function of D_b, as well as ΔT. In our simple example, ΔT goes steeply through zero in the region of the minimum of χ^2.

Of course, for understanding the origin of Kuhn, Libbrecht & Dicke's (1988) latitudinal temperature bands, as well as solar irradiance variation, it is essential to determine the temperature change at the base of the solar photosphere during the solar cycle. As we mentioned, there is a contradiction regarding the relative sign of temperature and magnetic field changes (Spruit and Brüggen 2000). In our illustrative example, we get $\Delta T < 0$ for $\Delta B > 0$ corresponding to the χ^2 minimum. This would mean that the active latitudes are cooler. However, from the left panel of Fig.6, it is clear that we are not far from the place where ΔT changes sign, so we must regard the illustrative finding as inconclusive. We see in Fig.6 that the minimum of χ^2 is much better determined from the centroid data. In the future, we will consider the condition of thermal equilibrium in the sun's outer layers in order to make use of these data to constrain the surface averaged temperature change. We propose a more thorough analysis of the MDI data, as well using constraints on ΔB_r from BBSO Ca II K and the BBSO Solar Disk Photometer data. Further, we propose to approach the problem on the theoretical side by considering and applying the condition of thermal equilibrium. Naturally, for the irradiance problem we have to know $(\frac{\Delta T}{T})_0$.

A supplementary approach to the problem of the relationship between activity and temperature that we propose is an analysis of multi-color photometry data from the Solar Disk Photometer (SDP) at BBSO. Ken Libbrecht (Caltech) designed the SDP to improve upon the Mt. Wilson Limb Photometer by allowing detection of not just the limb, but a number of successive annuli on the solar disk – ranging from the limb, to as small as about half the disk radius. With these data, we reproduce Kuhn's temperature bands (as did Woodard and Libbrecht 1993), but since we have superior information about limb darkening it should be easier to disentangle the various contributions to the signal. In Fig.7, we demonstrate the

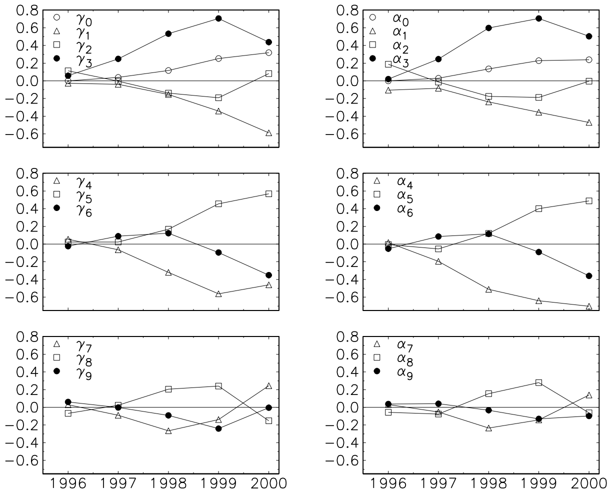

Figure 7. Right panels show the Legendre polynomial expansion of the photometric signal following the analysis in Goode et al. (2001). Left panels show the corresponding γ's. The latter were averaged to the time frame of the α's.

correlation between the Legendre coefficients from the photometer and from seismology, Goode et al. (2001). The prospect for determining temperature follows from the fact that we have three-color data, and the geometrical effect may be handled from information on the limb darkening. The fact that we have both α_0 and γ_0 is significant because it provides a tool to link irradiance variation to luminosity variation.

Finally, let us remark that in the discussion of the seismic method of field and temperature determination, the kernels were regarded as being ℓ-independent. But our formalism is, in fact, more general. In particular, it is valid for solar f-modes. Furthermore, for the low-order Legendre polynomial distortion, the formalism is useful for assessing the effect of a deeply buried magnetic field. We propose to use this more general formalism to set a reliable upper limit for the field near the base of the convective zone. Furthermore, with this formalism we can tell more about the field in the bulk of the convection zone, as Dziembowski, Goode & Schou (2001) indirectly inferred from the f-mode frequency changes.

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