Does the Sun Shrink with Increasing Magnetic Activity?

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ABSTRACT

We have analyzed the full set of SOHO/MDI f- and p-mode oscillation frequencies from 1996 to date in a search for evidence of solar radius evolution during the rising phase of the current activity cycle. Like Antia et al. (2000), we find that a significant fraction of the f-mode frequency changes scale with frequency; and that if these are interpreted in terms of a radius change, it implies a shrinking sun. Our inferred rate of shrinkage is about 1.5 Km/y, which is somewhat smaller than found by Antia et al. We argue that this rate does not refer to the surface, but rather to a layer extending roughly from 4 to 8 Mm beneath the visible surface. The rate of shrinking may be accounted for by an increasing radial component of the rms random magnetic field at a rate that depends on its radial distribution. If it were uniform, the required field would be ~ 7 KG. However, if it were inwardly increasing, then a 1 KG field at 8 Mm would suffice.

To assess contribution to the solar radius change arising above 4Mm, we analyzed the p-mode data. The evolution of the p-mode frequencies may be explained by a magnetic field growing with activity. Our finding here is very similar to that of Goldreich et al.(1991). If the change were isotropic, then a 0.2 KG increase, from activity minimum to maximum, is required at the photosphere, which would grow to about 1 KG at 1 Mm. If only the radial component of the field were to increases, then requirement for the photospheric field increase is reduced to a modest 60-90 G. A relative decrease in temperature of the order of 10^{-3} in the sub-photospheric layers, or an equivalent decrease in the turbulent energy, would have a similar effect to the required inward growth of magnetic field change.

The implications of the near-surface magnetic field changes depend on the anisotropy of the random magnetic field. If the field change is predominantly radial, then we infer an additional shrinking at a rate between 1.1-1.3 Km/y at the photosphere. If on the other hand the increase is isotropic, we find an expansion at a rate of 2.3 Km/y. In any case, *the rate of the sun's shrinking or expansion during the rising phase of activity is a few Km per year, and is probably undetectable by non-seismic means.*

Subject headings: Sun: radius — Sun: activity — Sun: oscillations — Sun: interior

1. Introduction

Measuring the sun's radius, and its variability, are significant, long-standing problems, especially in the context of understanding the cause of solar irradiance variations. Recently, it has been pointed out that helioseismology can provide a useful measure of the solar radius. Schou et al. (1997) and Antia (1998) showed that f-modes frequencies are good probes of the radius, and they inferred a value of the solar radius which is about 300 Km smaller than the one adopted in solar models at that time. The model values were based on a direct measurement of the sun's photospheric radius. The smaller radius has been confirmed by Brown and Christensen-Dalsgaard (1998) from many years of transit measurements using the Solar Diameter Monitor. The connection between the "true" solar radius and that inferred from f-modes is explained in §4.1.

Following the suggestion of Schou et al. (1997), Dziembowski et al. (1998, 2000) used f-mode data from the MDI/SOHO instrument to determine the evolution of the seismic solar radius through the rising phase of the present activity cycle. They reported statistically significant variations that showed no apparent correlation with activity measures. On the other hand, with GONG f-mode frequencies (ranging between 1.15 mHz to 1.45 mHz, or equivalently ℓ from 100 to 200), covering the rising phase of activity to the beginning of 1999, Antia et al. (2000) found a net decrease of about 5 Km in the solar radius. They attributed the difference with Dziembowski et al. (1998) to the latter's use of higher degree modes (up to ℓ=300). They pointed out the latter ℓ-modes are more likely to be effected by factors other than an evolving radius.

In the present work, we use oscillation data from SOHO/MDI covering 1996.3-2000.5. We first use f-mode data in an effort to infer a signal of radius change. We then modified our earlier analyses to consider other sources of the variations. Our analysis is preceded by an explanation of the meaning of the "seismic" radius inferred from f-modes. We then compare our results to those of Antia et al., and interpret the inferred rates in terms of magnetic field and temperature changes.

Our interpretation of p-mode frequency changes is predicated on the work of Goldreich et al. (1990) who analyzed BBSO measurements from the rise of the previous cycle (cycle 22). We use our inference on the behavior of the sub-photospheric layers to constrain radius changes arising there.

2. Frequency data from SOHO/MDI

In the present study, we use 19 MDI data sets containing centroid frequencies determined from measurements made between May 1, 1996 and June 21, 2000, with the break between June 16 and October 22, 1998, when there was no contact with SOHO. The sets are typically 72-days long, except those immediately before and after the break, which are shorter.

The sets contain between 112 and 203 f-mode centroid frequencies, with earlier sets having more data. The maximum ℓ- value is 300 and the minima range from 89 to 137. The number of p-mode frequencies range from 1589 to 1906. Again, the earlier sets are mode abundant. The ℓ-values of p-modes are in the (0–200) range. The difference in mode composition is not important in the case of p-modes. The number of overlapping modes is large enough for a detailed study of frequency changes. In the case of f-modes, the difference in the ℓ-range may be important, and therefore in our study of the solar radius changes, we used only modes with $\ell \leq 137$.

3. Inferences from f-mode frequency changes

3.1. Helioseismic radius

All helioseismic determinations of the solar radius to date have relied on the following asymptotic relation for f-modes frequencies,

$$\frac{\Delta\nu_\ell}{\nu_\ell} = -\frac{3}{2}\frac{\Delta R}{R}. \tag{1}$$

Antia et al. (2000) pointed out that using this relation for modes with ℓ extending up to 300, as Dziembowski et al. (1998) did, is not justified because of significant departures from $\nu \propto R^{-1.5}$ are present in higher ℓ's. The departure increases with ℓ, which as Brown (1984) first suggested could be accounted for as an effect of turbulence in the upper convective zone. Detailed models of this effect have been developed by Murawski & Roberts (1993a, 1993b). (For the most recent work on the subject see Mędrek & Murawski 2000). However, surface magnetic fields also may have significant effects on f-mode frequencies (Evans & Roberts, 1990; Jain & Roberts). With these two sources of perturbation to f-mode frequencies, we must contemplate solar cycle changes beyond that of a simple radius change. The relative contribution of the near-surface changes are expected to increase with ℓ, because such changes should be inversely proportional to mode inertia, I_ℓ, which sharply decreases with ℓ.

There is another problem in applying Eq.(1) in a search for the radius variations correlated with activity. This problem follows from the fact that the induced modifications are quite non-uniform, and each f-mode has it is own radius, R_ℓ, which is given by

$$R_\ell = \left(\frac{1}{I_\ell}\int r^{-1/3} dI_\ell\right)^{-1/3}. \qquad (2)$$

With this definition, we get from the variational principle for oscillation frequencies

$$\nu_l = \frac{1}{2\pi}\sqrt{(L-2)\frac{GM}{R_\ell^3}}, \qquad (3)$$

where $L = \sqrt{\ell(\ell+1)}$. This is a very accurate expression. The relative departures from equality range from 2×10^{-4} at $\ell = 100$ to 8×10^{-5} at $\ell = 300$. In a linear approximation in terms of the $(R - R_\ell)$, this is the same as the formula obtained by Gough (1993).

For high degree modes, the f-mode radii are close to the solar radius. The values of R_ℓ/R range from 0.9883 at $\ell = 100$ to 0.9946 at $\ell = 300$. While we have $R_\ell \approx R$, a corresponding approximation for ΔR_ℓ is quite problematic. When the f-mode frequencies were used to refine the value of the radius for modeling the sun, we could expect an approximate, homologous relation, $R_\ell \propto R$. But such a relation cannot be expected in the case of the activity induced changes, which we believe to be confined to the outermost part of the sun. If the data show that $\Delta\nu_\ell \propto \nu_\ell$, as Antia et al.(2000) found, then the simplest interpretation is that, indeed we have ΔR_ℓ being constant over the range of considered ℓ-values. Then, the inferred value of ΔR in Eq.(1) refers to the range of depths beneath the photosphere corresponding to the range of ℓ's in the data sets. Antia et al. (2000) used modes in the 100–200 range, which translates to 10–6 Mm in depth. Their finding implies that this layer was moved downward by about 5 Km during the two years they considered. The truth is, with these data, we cannot say anything about what happened in the layers above. Thus, we have no information about the evolution of the surface radius of the sun.

3.2. Formal determination of the rate of shrinking from f-modes

To account for the effect of the near-surface changes on f-mode frequencies and possible differential changes, we modified Eq.(1) into

$$\Delta\nu_\ell = -\frac{3}{2}\frac{\Delta R_f}{R}\nu_\ell + \frac{\Delta\gamma_f}{I_\ell}, \qquad (4)$$

where ΔR_f denotes the radius change inferred from a particular set of f-modes. For the calculation of I_ℓ, we adopted the following normalization of the eigenfunctions

$$(\xi_r)_{\rm phot} = \frac{2\times10^4}{\sqrt{\bar\rho R^5}} Y_\ell^m, \qquad (5)$$

where ξ_r denotes radial displacement of the fluid element. With such a normalization, the values of $I_\ell(\nu)$ are of the order of unity for p-modes in the 2–4 mHz range. For the f-modes, the I_ℓ values are significantly larger (see Table 1).

We determined ΔR_f and γ_f by a least-squares fitting. In Fig. 1, we show values of $\Delta\nu_\ell$ for selected data sets. The first two sets were obtained at solar minimum. The 1999.4 set was taken near the middle of the phase of rising activity, and the last is at the current maximum. Here, Δ denotes the difference between the solar data and the reference model. The reference solar model is that of Christensen-Dalsgaard et al.(1995). It has the same radius used by Dziembowski et al. (1998). That is, the model uses the "old", too small value of the solar radius (not that of Brown and Christensen-Dalsgaard 1998), and this is why the frequency differences are rather large. A small difference in the reference model radius is inconsequential for the inferred temporal changes.

We assumed that $\Delta\gamma_f$ is ν-independent, and as we see in Fig. 1, the fit is good. The χ^2's vary from 1.2 (2000.4 set) to 1.84 (1996.4). A significantly worse fit ($\chi^2 = 3.55$) was found only from the 1998.9 set, which was the first one taken after the recovery of SOHO. We also tried a fit to γ_f including a low-order polynomial depending on frequency, but this did not improve the fit.

We see the departure from the linear relationship implied by the the radius adjustment sharply increases with ν. Antia et al. (2000) considered only modes with $\nu < 1.44$ mHz, and it seems that the departure from a straight line is still small. However, this is somewhat misleading because we used a model with much too large a radius. As we shall see in Table 1, at the level of changes of a few nanohertz (i.e. radius changes of a few Km), the difference is quite significant. We emphasize that high ℓ-modes are important because with increasing ℓ, R_ℓ approaches the solar radius. For such modes, including γ_f is essential, which implies that we have to rely on Eq.(4) rather than Eq.(1). With Eq.(1), we get a much poorer fit ($\chi^2 = 4.4 - 16.5$) and the correction to the solar radius is larger by some 20 Km. This illustrates the trade-off – increasing ℓ moves us closer to the surface, but such high ℓ's are more strongly contaminated.

Our data sets from solar minimum contain some modes with ℓ as low as 90. To avoid spurious changes in ΔR_f, following from the difference in the minimum values of R_ℓ, we eliminated all the data for which $\ell \leq 137$.

In Fig. 2, we show the variations of the solar radius and γ_f inferred from f-modes from the truncated data sets. The rise of the current activity cycle began in 1997.4 which was marked by a sharp rise of the seismic activity indicators (Dziembowski et al. 1998). These are seen in Fig. 3. That is why we choose 1997.4 to begin our linear fits. We have no explanation for the relatively large fluctuations in ΔR_f and in $\Delta\gamma_f$ between 1996.4 and 1997.0. They have no counterpart in the variations seen in the p-modes. For comparison, we also show the result obtained when the γ_f-term is ignored. There is a difference, but not as large as one might anticipate by looking at Fig.1. The rate of radius decrease is only insignificantly higher than in our standard version, and the error is larger.

In detail, we found from our linear fit, with the γ_f,

$$\frac{dR}{dt} = (-1.51 \pm 0.31) \quad \mathrm{Km/y}, \tag{6}$$

and without the γ_f-term,

$$\frac{dR}{dt} = (-1.82 \pm 0.64) \quad \mathrm{Km/y}.$$

Table 1: Contributions to f-mode frequency shifts during the rising phase of cycle 23

ℓ	ν_ℓ[mHz]	I_ℓ	$\Delta\nu_R[\mu\mathrm{Hz}]$	$\Delta\nu_\gamma[\mu\mathrm{Hz}]$
100	1.02	381	0.010	0.0012
130	1.15	165	0.011	0.003
200	1.43	39.	0.014	0.012
300	1.74	9.4	0.017	0.050

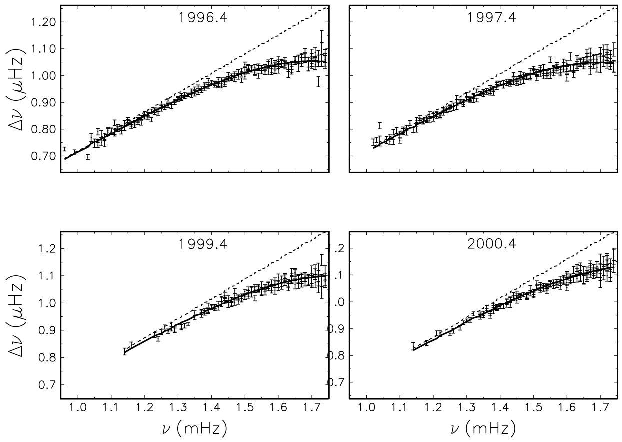

Fig. 1.— Differences between measured and calculated f-mode frequencies. The error bars show estimated standard deviations of measured values. The dates correspond to the center of the individual 72-day long measurement periods. The solar model was calculated assuming $R_\odot = 695.991$ Mm. The solid line represents the fit to Eq. (2). The dashed straight line represents the part attributed to the difference between the solar radius and that adopted in the model.

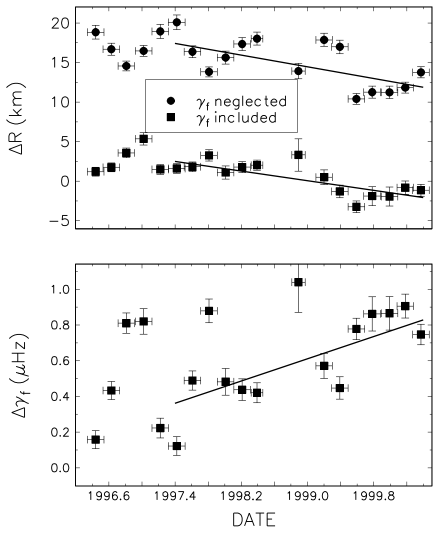

Fig. 2.— *Upper panel*: Variation of solar radius between 1996.4 and 2000.4 inferred from f-mode frequencies with and without the γ_f-term. Two straight lines represent linear fits to the data starting from 1997.4 when the rise of cycle 23 began. *Lower panel*: Corresponding variation of γ_f, which describes remaining near-surface contribution to f-mode frequency variations.

The values are similar to those found by Antia et al. (2000). To make a closer comparison, we truncated our data sets at $\ell = 200$, and then we found

$$\frac{dR}{dt} = (-1.80 \pm 0.38) \quad \mathrm{Km/y}.$$

Having in mind that we still miss modes between $\ell = 100$ and 137, it is fair to say that there is no disagreement between our findings and theirs, implying that at a depth of from 6 to 10 Mm the sun shrank by some 4 to 6 Km during the rising phase of this activity cycle.

How reliable is this finding? The main concern is the role of the near-surface perturbation and the cross-talk between the two terms in Eq.(4). In the lower panel of Fig. 2, we show the γ's. The linear fit for γ, which is visibly poorer, yields

$$\frac{d\gamma_f}{dt} = (0.180 \pm 0.051) \quad \mathrm{Km/y}. \qquad (7)$$

The relative contribution of the two terms to overall f-mode frequency variations depends on ℓ. In Table 1, we compare these two contributions, denoted by$\Delta\nu_R$ and $\Delta\nu_\gamma$ for selected ℓ-values. The increasing role of $\Delta\nu_\gamma$ is a consequence of decreasing mode inertia. It should be noted that $\Delta\nu_\gamma$ yields an appreciable contribution to $\Delta\nu$ even for modes with $\ell \leq 200$. Caution is necessary, but we will proceed further assuming that the effect is indeed real.

3.3. Accounting for the rate of shrinkage

Even as small as it seems, a shrinking of the sun's radius during the rising phase of activity is not easy to explain. We expect that magnetic activity can cause structural changes only in the outermost part of the sun. Thus, we have $r \approx R$ and

$$\Delta r = r - r_0 \approx -\int_{r_b}^{r_0} \frac{\Delta\rho}{\rho} dr_0, \qquad (8)$$

where r_b is the radius at the bottom of the layer perturbed by activity, and r_0 is the radius at a specified fractional mass, M_r/M, at activity minimum and $\Delta\rho$ denotes the horizontally averaged change of density. We obtain a more revealing form of Eq.(7) by expressing $\Delta\rho$ in terms of the averaged entropy and magnetic field changes. At the depth of interest, we may use the equation of state for an ideal gas. For the horizontally averaged gas pressure in the presence of a random magnetic field we have, after Goldreich et al. (1991),

$$\Delta P_g = -\Delta(\beta P_m), \qquad (9)$$

where

$$P_m = \frac{\overline{B_h^2} + \overline{B_r^2}}{8\pi}$$

is magnetic pressure and

$$\beta = \frac{\overline{B_h^2} - \overline{B_r^2}}{8\pi P_m}$$

is a measure of the statistical anisotropy of the field.

With the use of thermodynamical relations, we determine

$$\Delta r = \int_{r_b}^{r_0} \left[\frac{1}{\Gamma_1} \frac{\Delta(\beta P_m)}{P_g} + (-\rho_T)\frac{\Delta S}{c_p} \right] dr_0, \qquad (10)$$

where ρ_T denotes the logarithmic derivative of density at constant pressure. The remaining thermodynamical quantities have their standard meanings. At the relevant depths, the gas is nearly ideal. Thus, we may use $\rho_T = -1$, $1/\Gamma_1 = 0.6$, and

$$\frac{\Delta S}{c_p} = \frac{\Delta T}{T} - 0.4\frac{\Delta P_g}{P_g}.$$

The irradiance from an active sun is higher than average. If the same is true about luminosity then we should have $\Delta S < 0$. Hence, a negative contribution to Δr. However, this must be very small. If ΔS refers to the whole convective zone then a 10^{-3} luminosity increase translates to an annual decrease in $\Delta S/c_P$ of 10^{-7}. Another possibility is an increase in the superadiabatic gradient, $\nabla_{\rm con} - \nabla_{\rm ad}$, but this seems unlikely too. The annual decrease of $\Delta R_f = 1.5$ Km refers to the layer of $r/R = 0.988 - 0.995$. Thus, ΔR_f must arise mostly beneath $r = R_{137} = 0.988R$. At this depth, according to a mixing-length model, $\nabla_{\rm con} - \nabla_{\rm ad} \approx 2 \times 10^{-4}$, which rapidly decreases going inward. We would need an order of magnitude increase in the superadibatic gradient to account for our rate of shrinking.

A more acceptable explanation would be a variation in the magnetic field. The consequences of a magnetic field increase depend on β. For a purely radial field ($\beta = -1$), the increase implies contraction. For an isotropic field ($\beta = 1/3$) the increase implies expansion. The field geometry implying the minimum increase to account for the rate of the shrinking corresponds to $\beta = -1$. Then, we have

$\Delta < B >_{\rm rms}= \sqrt{\Delta(B_r^2)}$, and assuming a constant rate across the lower convective zone, we infer

$$\frac{d< B >_{\rm rms}}{dt} \approx 7.3 {\rm KG/y}.$$

The value at R_{137} may be reduced, for instance, to 1 KG/y if one allows an exponential increase of the rate to about 40 KG/y at the base of the convection zone.

Thus, what we have inferred from the f-mode frequency change is a non-trivial constraint on the internal magnetic field change. Let us note that if the field increase were predominantly isotropic, we would see an expansion rather than a contraction. The field increase inferred from the residual (after removing the near surface contribution) part of the p-mode frequency change was about 60KG at 25-100 Mm (Dziembowski et al., 2000). This high value could be consistent with the shrinking rate only if β is close to zero, that is if the field is essentially force-free, which is not a likely possibility. Thus, we are now skeptical about the reality of that large field change we reported earlier.

Our inference regarding the solar radius change is limited by the lack of accurate information about what happened in the outer 4 Mm of the solar interior. This is the region where we may expect the largest activity induced variations for two reasons. First, the rapid decline of gas pressure and second, the thermal structure of this layer is more susceptible to changes in the efficiency of the convective energy transport induced by the field changes. The f-mode data we have at hand provide some information about changes in this layer through the γ_f. Similar, but much accurate information is available in the p-mode data, which we now consider.

4. Inference from p-mode frequency changes

4.1. The near-surface source of the p-mode frequency changes

The p-mode spectrum of MDI frequency data is about 13 times richer than that for f-modes. Unfortunately, p-modes are not directly useful for determining changes in the solar radius. The simple relation, $\nu \propto R^{-1.5}$ would be valid for p-modes only if the changes were homologous throughout the whole sun. This is far from the truth for the changes we are considering here. However, from p-modes one may make a much more precise determination of the near-surface perturbation. For p-modes, we call it $\Delta\gamma_p$, and it describes frequency changes caused by a variable perturbation localized near the surface. In the present application, however, taking into account the ν dependence is now required for an accurate fit. We express the dependence in the form of a Legendre polynomial series with argument

$$s = \frac{\nu - (\nu_l + \nu_h)/2}{\nu_h - \nu_l},$$

where ν_l and ν_h denote the lowest and the highest frequencies in the data set. Thus, we write

$$\Delta\nu_{\ell,n} = \frac{1}{I_{\ell,n}} \sum_0^J \Delta\gamma_{p,j} P_j(s). \tag{11}$$

Here Δ is with respect to the 1996.4 data set. The number J was increased until $\gamma_{p,0}$ stabilized within the errors and χ^2 stabilized. This occurred for $J = 2$. In Fig. 3, we plot $\Delta\gamma_{p,0}$ for $J = 0$ and 2. Variations of γ_p are indeed much more accurately determined than those of γ_f. For $J = 2$ we find the rate

$$\frac{d\gamma_{p,0}}{dt} = (0.149 \pm 0.008) \quad {\rm Km/y}. \tag{12}$$

The dependence of $\gamma(\nu)$ yields an important constraint on the localization of the source of solar cycle variations in p-mode frequencies.

Following Goldreich et al. (1991), we link the frequency change to the change of the mean squared magnetic field and a Lagrangian change of a single thermodynamic parameter. For the latter, we prefer to use temperature rather than entropy. From Eqs.

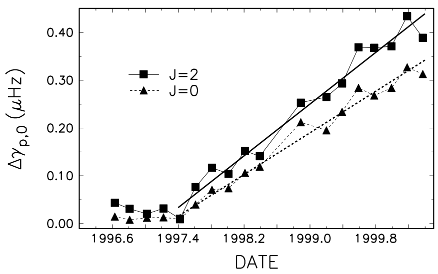

Fig. 3.— Variation of the mean value of γ with two versions of its polynomial dependence inferred from p-mode frequencies. As in the case of f-modes shown in Fig. 2, the linear fit corresponds to the data from 1997.4 when the rise of cycle 23 began. The error bars would be within the symbols.

(14) and (15) of Goldreich et al., we get the following expression for the change of γ_p,

$$\Delta\gamma_p = \frac{1}{8\pi^2\nu}\int d^3\vec{x}|\mathrm{div}\vec{\xi}|^2\{P\Gamma_1(1+\Gamma_\rho)\rho_T\frac{\Delta T}{T} + [1+\Gamma_1(\Gamma_P+\Gamma_\rho\rho_P) - \beta(\Gamma_1 - 1 + \Gamma_1\rho_P)]\Delta P_m\}. \qquad (13)$$

Here, we denote by Γ_P and Γ_ρ, the logarithmic derivatives of Γ_1. The ideal gas equation cannot be used in these layers, where most of the contribution to $\Delta\gamma_p$ arises.

Goldreich et al. (1991) explained the p-mode frequency changes during the rising phase of cycle 22 in terms of magnetic field and temperatube changes, with former being dominant and causing the frequency increase. They invoked a chromospheric temperature increase to explain the reversal in the increasing trend in $\Delta(\nu)$. We do not see such a trend in our data. Thus, as a first guess we interpret $\Delta\gamma_p$ in terms of magnetic field changes. Later, we will discuss other sources of the p-mode frequency changes.

We considered two values of β, -1 and 1/3, and the following form for the depth dependence of magnetic field increase

$$\Delta <B>_{\rm rms} = \begin{cases} B_b & \text{if } D \geq D_b \\ B_b\left(\frac{D-D_b}{D-D_m}\right) & \text{if } D_b < D < D_t \\ \Delta <B>_{\rm rms}(D_t) & \text{if } D \geq D_t \end{cases} ,$$

where $D_m = -0.485$Mm denotes D at the temperature minimum, and B_b, D_b, and λ were determined by fitting the three terms in the series (11). For D_t we adopted either D_m or 0.

In Fig. 4 we show two examples of the field's changing behavior that would be consistent with the observed γ's, and compare them with two cases that are clearly inconsistent. One of these two cases is a depth independent increase, and the other is an example of the field gradually increasing to about 3 KG at 8Mm. In all four examples, we used $\beta = -1$. We see that indeed the $\gamma_p(\nu)$ provides a strong constraint on the localization of the source of frequency changes, but clearly not a unique answer. An equally good fit was obtained with the choice $\beta = 1/3$. Data on the three models of the magnetic field change fitting $\Delta\gamma_p(\nu)$ data are given in Table 2. The result for $\beta = 1/3$ is not significantly different from that found by Goldreich et al. (1991). To explain the p-mode frequency increase between minimum and maximum, we require

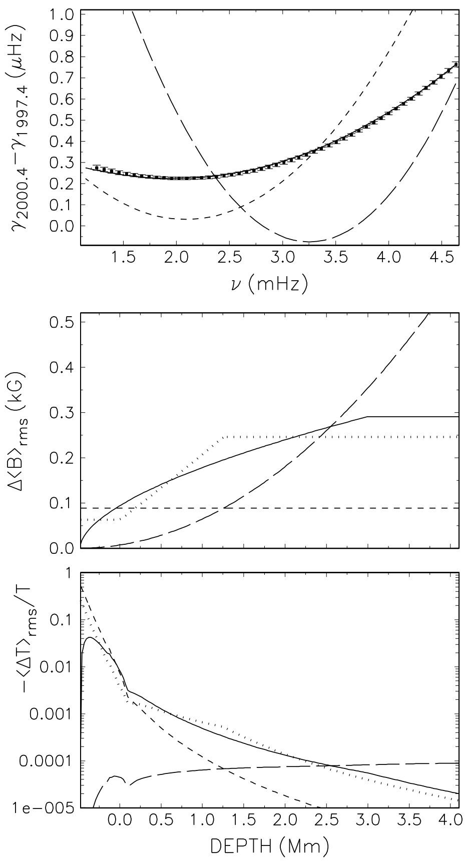

Fig. 4.— In the top panel, points with the error bars represent $\Delta\gamma_0(\nu)$ inferred from p-mode frequency difference between 2000.4 and 1997.4. The lines correspond to various distributions of the averaged magnetic field, shown in the middle panel. The solid and dotted lines are within the error bars in the top panel. The bottom panel shows the relative temperature decreases required to cause similar frequency shifts as the corresponding magnetic field increases

an increase of the rms magnetic field growing from 0.2 KG in the photosphere to 0.84 KG at 4.25 Mm. The corresponding numbers of Goldreich et al. are 0.25 and 1 KG.

In the bottom panel of Fig. 4, we plot the relative temperature changes, which gives the same local contributions to $\Delta\gamma_p$ as the corresponding changes in the magnetic field. We see that the required change of temperature is unacceptably large in the atmospheric layers. However, in sub-photospheric layers we cannot exclude the rms $\Delta T/T$ at the 10^{-3} level. Such a temperature decrease would be a significant contributor to the observed frequency increase. Brüggen and Spruit (2000) argue that one expects a lower subsurface temperature from an increasing magnetic field, and that the effect should be searched for by means of helioseismology. A contribution from temperature decrease would lower the requirement for the magnetic field increasu in the sub-photospheric layers.

Yet another potential contbibutor to the frequency increase is a decrease in the turbulent velocity. Roughly, the relative change in the turbulent velocity, $\Delta v_t/v_t = q$, has the same effect as a relative temperature change $\Delta T/T = 0.5q\mathcal{M}^2$, where $\mathcal{M}$ is the turbulent Mach number. In the sub-photospheric layers, $\mathcal{M}$ is in the 0.1–1 range. Thus, the effect may be significant, and we may expect a decrease in v_t because the magnetic field should inhibit convection.

4.2. Shrinking or expanding of the outermost layers

In Table 2, we provide the values of the contribution to the rate of the photospheric radius change due to the magnetic field increase inferred from the γ_p changes. We emphasize that the rate does not refer to photosphere but to the mass point corresponding to the unperturbed (solar minimum) photosphere and that the value does not include the part that was inferred from f-mode frequency changes.

Solar photosphere is defined as a surface of specified optical depth $\tau_{\rm phot} = M_{\rm phot}\bar\kappa$, where $\bar\kappa$ is the mean opacity in the atmosphere. Thus, the rate of movement of the photosphere is given by

$$\frac{dR_{\rm phot}}{dt} = \left(\frac{dR}{dt}\right)_{\rm phot} - \left(\frac{d\bar\kappa}{dt}\frac{dr}{\bar\kappa}\right)_{\rm phot}, \tag{14}$$

An estimate shows that the second term may not be negligible if $(\Delta T/T)_{\rm phot} \sim 10^{-3}$.

The total value of $dR/dt_{\rm phot}$ may be estimated as the sum of -1.5 Km/s inferred from the f-mode data and one of the values inferred from p-modes data shown in Table 2. These are model dependent. We do not expect that by including the nonmagnetic contributors to $\Delta\gamma_p$, we would infer rates significantly beyond the range of values quoted in this table. We note that the net effect may imply both contraction and expansion. Possible net values of $dR/dt_{\rm phot}$ range from -3 to 1 Km/y.

Finally, we point out that there is no contradiction between our inferences from f- and p-mode frequency changes. The effect of the field increases needed to account for the dR/dt_f value have a negligible effect on p-modes frequencies, if the outward decrease of $d < B >_{\rm rms} /dt$ from the bottom of the convective zone is steep enough.

Table 2: Inference from p-mode frequency changes between 1997.4 and 2000.4

β	λ	D_b	D_t	B_b	B_t	$(dR/dt)_{\rm phot}$
-1	0.575	3.00	-0.485	0.29	0.062	-1.3 Km/y
-1	1.15	1.27	0	0.25	0.094	-1.1 Km/y
1/3	0.623	4.25	-0.485	0.84	0.20	2.3 Km/y

5. Conclusions

Results of our analysis of f-mode frequency confirms the evidence, first found by Antia and Basu (2000), for a contraction of the sun's outer layers during the rising phase of the magnetic activity. The rate we determine is 1.5 Km/y and is only somewhat different than found by our predecessors. We pointed out, however, that there may be another interpretation for the observed frequency variation. Further, we stressed that the rate does not refer to the surface radius, but to the layer at 4 – 8 Mm depth below the photosphere. In spite of the fact that the dispersion relation for high-degree f-modes approaches that for the surface gravity waves, the two types of modes are essentially different. While the latter are discontinuity modes which see the same gravity for each horizontal wave number, the f-modes see different effective gravities depending on ℓ.

The rate of shrinking is most easily explained as resulting from the rise of the radial component of the random magnetic field beneath a depth of 8 Mm. There is an integral constraint on the magnetic field which may be the most important finding from the data on f-mode frequency changes. To account for the shrinking rate, we need an increase in the radial component of the random magnetic field with a modest annual rate. An isotropic increase would imply an expansion.

The p-mode frequency change may be accounted for in terms of magnetic field changes. Our analysis was based on the formalism of Goldreich et al. (1991), and we found similar implications regarding the required field increase as these authors, who analyzed BBSO data from the previous solar maximum. In particular, the increase must be larger below the photosphere than in the atmosphere, if this is the sole effect causing p-mode frequency changes. We pointed out, however, that a temperature decrease and/or decrease of turbulent velocity in sub-photospheric layers could be significant contributors to the frequency decrease. Depending on the field anisotropy, the changes in the outermost layers may lead to additional shrinking or to net expansion.

In any case, the radius changes implied by the f- and p-mode frequency increase from the last minimum to current maximum is small compared to the errors. Our estimate of the total change ranges from -9 to 3 Km. This range is well within the uncertainty of the direct radius measurments of Emilio et al. (2000), who used SOHO intensity data. The small radius change we determined here would cause a negligible change in the solar irradiance.

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