Evidence for onset of solar activity from SOHO/MDI seismic data

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Abstract

We analyzed time changes in frequencies determined with the SOHO/MDI instrument. The data were divided into five 72-day sets covering the period from May 1, 1996 to April 25, 1997. We found a significant trend in the behavior of the a_4 and a_6 coefficients, which reflect a decrease in the P_4 distortion and an increase in that from P_6. This trend is the same as seen in the BBSO data (Libbrecht and Woodard, 1990) between 1986 and 1988, i.e. at the onset of the previous activity cycle. There is no apparent trend in a_2.

The centroid frequencies, $\bar{\nu}_{\ell,n}$, as already reported by Kosovichev et al. (1998), exhibit small non-monotonic variations. The relative differences among the solar radius inferred from the f-mode frequencies from the five sets (at most 5×10^{-6}) are formally significant, but again there is no monotonic trend.

1. Introduction

The asymmetric part of the fine structure in the p-mode spectrum of solar oscillations varies in a systematic way through the solar cycle, Kuhn (1988, 1989) and Libbrecht and Woodard (1990). It is evident that the changes are associated with the surface temperature bands reported by Kuhn, Libbrecht and Dicke (1988). As well, Woodard and Libbrecht (1991) found a strong correlation between oscillation frequency changes and solar surface magnetic variations from monthly averages of their data. One conspicuous feature is the persistent manifestation of a P_4 distortion in the splittings. The origin of this latter behavior as well as the temporal variation of the frequencies is unclear. However, the data to date have an incomplete coverage of the cycle. In addition, we have no basic understanding of the correlation among the various manifestations of solar asphericities and the variation of the mode multiplet frequencies.

Centroid frequencies were also shown to vary through the solar cycle. This has been demonstrated for both modes of moderate ℓ, Libbrecht and Woodard (1990), and low ℓ modes (Anguera-Gubau, et al. ,1992) and Elsworth et al., 1994). Dziembowski and Goode (1996) showed that all these changes are consistent with a near surface perturbation. That is, there is no evidence for solar cycle variations of the frequencies arising from deeply buried changes.

The SOHO/MDI instrument has the capability of following various forms of the asphericities. In particular, Kuhn, et al. (1998) studied the asphericities in brightness, and they detected a significant hexadecupole (P_4) distortion. The same instrument measures oscillation frequencies.

2. Frequency data from SOHO/MDI

The data treated here are five 72-day sets covering (1) 5/1/96-7/11/96, (2) 7/12/96-9/21/96, (3) 9/22/96-12/2/96, (4) 12/3/96-2/12/97, and (5) 2/13/97-4/25/97. They contain centroid frequencies $\bar{\nu}_{\ell,n}$ and 36 splitting coefficients a_k. Each of the sets has data for 1348 p-mode multiplets (covering ℓ ranging from 2 through 184) and 33 f-mode multiplets. For each p-mode multiplet, the splittings are defined by

$$\nu_{\ell,n,m} - \bar{\nu}_{\ell,n} = \sum_{k=1} a_k \mathcal{P}_k^{\ell}(m), \tag{1}$$

where $\mathcal{P}$ are are orthogonal polynomials (see Ritzwoller and Lavely 1991 and Schou, et al. 1994). The remaining symbols in eqn. (1) have their usual meanings. This representation ensures that the $\bar{\nu}_{\ell,n}$ are a pure probe of the spherical structure while the a_{2k} – the even-a coefficients – are a pure probe of the distortion described by corresponding $P_{2k}(\cos\theta)$ Legendre polynomials. We have

$$\int_0^{2\pi}\int_{-1}^{1} |Y_\ell^m|^2 P_{2s} d(\cos\theta) d\phi = C_{k,\ell}\mathcal{P}^\ell_{2k}(m), \tag{2}$$

where

$$C_{k,\ell} = (-1)^k \frac{(2k-1)!!}{k!}\frac{(2\ell+1)!!}{(2\ell+2k+1)!!}\frac{(\ell-1)!}{(\ell-k)!}, \tag{3}$$

In a representation of the splitting in terms of $P_k(\frac{m}{L})$, where $L=\ell$ or $\sqrt{\ell(\ell+1)}$, which was the standard some years ago, the association between a_{2k} and $P_{2k}(\cos\theta)$ was only valid asymptotically for $\ell \gg k$. The antisymmetric (about the equator) part of distortion is undetectable by means of a seismology based on eigenmode frequencies.

We separate the analysis of p-modes and f-modes because they have vastly different properties in the outer layers. Our inferences from the f-modes only concern possible variations in the seismic solar radius, and we postpone disussion of these modes to Section 6. Here, we only consider even-a coefficients and centroids of the p-modes.

The even order splitting coefficients in the five data sets were fit to the following formula

$$a_{2k,\ell,n} = a_{2k,\ell,n;\mathrm{rot}} + C_{k,\ell}\frac{\gamma_k}{I_{\ell,n}}, \tag{4}$$

where the first term on the r.h.s. represents the effect of centrifugal distortion which we calculate following the treatment of Dziembowski and Goode (1992). Since our plan is to study temporal variations in the asymmetry, we, therefore, eliminate the constant distortion of rotation from the γ's in eqn. (4). The $I_{\ell,n}$ denote mode inertia. They are evaluated assuming all modes have the same value for the radial displacement at the base of solar photosphere. Adopting a value of 10^4 for the displacement, we have $I_{20,19} \approx 1$. The inertia decreases with increasing ℓ or n. The same expression has been used to represent centroid changes, however, in this case we can determine only a relative γ_0. We choose SET 1 as the reference.

The $I^{-1}_{\ell,n}$ factor accounts for most of the ℓ and frequency dependence in the splitting coefficients. This was given by Libbrecht and Woodard (1990) as evidence that the asphericity responsible for the even-a coefficients is localized well above the lower turning point of modes in the sample i.e. close to surface. Dziembowski and Goode (1991) were able to determine a significant ν dependence of the γ_1 and γ_2 coefficients from the same data, and showed that it helps to determine the depth of the relevant asphericity. In general, we should also expect an ℓ-dependence of the $\gamma's$. We shall return to this problem later. First, we present the results of fitting constant γ's to each of the five sets of the even-a coefficients from the SOHO/MDI instrument.

In Figure 1, we show the first 12 γ's. We see that the errors rapidly increase with the order, k. We will focus on the first four coefficients in the expansion. The changes found in some higher order γ's are formally significant, but we are reluctant to interpret them. We do not see a significant trend in either γ_0 or γ_1. The frequency differences have already been presented by Kosovichev, et al (1998) in a different form. The fact that frequencies in the SET 1 are systematically lower than in the remaining sets has been already reported (Kosovichev, et al. 1998). However, they reported no apparent solar cycle variation.

The absolute values of γ_2 and γ_3 are much larger than those for γ_0 and γ_1. The first term which describes the P_4 distortion was dominant in 1996. The corresponding distortion has been found by Kuhn, et al. (1998) in their intensity measurement with the SOHO/MDI instrument. P_2 was the dominant distortion is their measurement, and was consistent with centrifugal stretching. In particular, if we represent their result for the asymmetry in terms of the fractional radius change in the Sun, we have

$$\frac{\delta R}{R_{\odot\,A}} = [(-5.41 \pm 0.46)P_2 + (1.43 \pm 0.56)P_4] \times 10^{-6}.$$

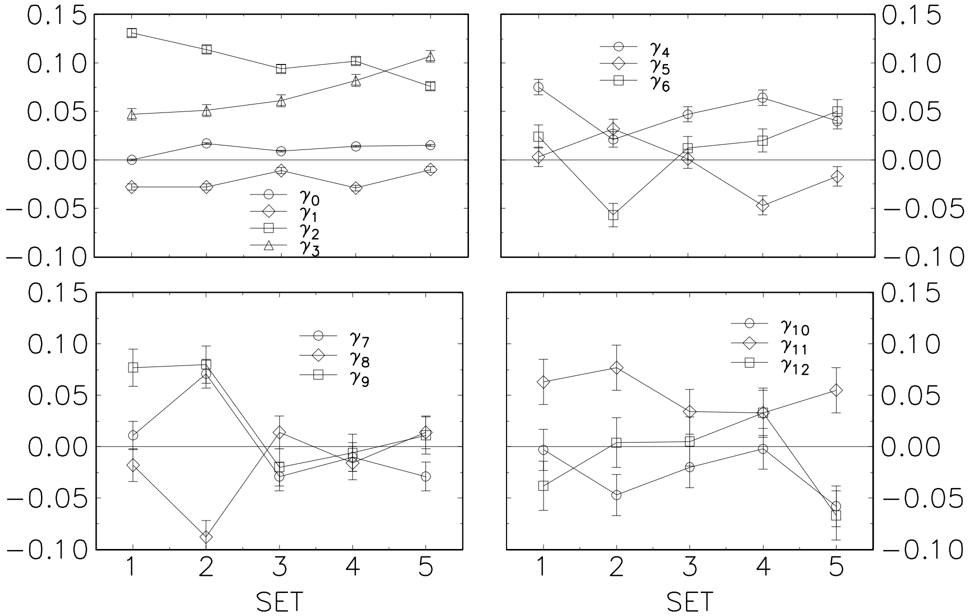

Fig. 1. Mean γ's (in μHz from from the five SOHO/MDI sets.

Roughly, a seismically determined differential rotation, using the rotation law in Dziembowski and Goode (1992) implies a value of

$$\frac{\delta R}{R_{\odot}}_{DR} = [-6.0P_2 + 0.6P_4] \times 10^{-6}.$$

Thus, after removing the part of the asymmetry due to rotation, only the P_4 distortion is significant at the 1σ level. We stress that we removed the effect of centrifugal distortion from our γ's. The effect is 0.037 μHz for γ_1 and about ten times smaller for γ_2 and is completely negligible for γ_3 and higher order terms. Note that the size of centrifugal stretching removed from γ_1 is smaller than γ_2 or γ_3. In detail, we find that the P_6 term is dominant in SET 5. This set was obtained contemporaneously with that of Kuhn, et al., but they did not report for a P_6 distortion. This might be due to the relatively larger noise level in the intensity data. The relative accuracy in determining the asymmetry serves to emphasize the broad utility of the seismic data.

3. Comparison with earlier results

The trend seen in γ_2 and γ_3 is the same as at the beginning of the rising phase of the previous activity cycle. We note that the data from the previous rising phase covered a similar frequency range, but they were restricted to lower ℓ values. The maximum value of ℓ in the BBSO data (Libbrecht and Woodard, unpublished) is 140, while in LOWL data (Tomczyk, private communication) it is 100. Thus, to make a meaningful comparison, we have to examine the behavior of the γ's with changes in the ℓ composition of the data−−in particular, the maximum value of ℓ in the truncated sets. Fig. 2 shows an example. We show there the effect of truncation of selected data sets on the inferred values of γ_2 and γ_3. The values are the largest in the two sets selected, but the patterns are the same for the other original sets. We see that there is a difference in the behaviors−−the value of γ_2 is insensitive to $\ell_{\max}$, whereas the value of γ_3 montonically increases with it. The behavior of γ_1 is similar to that of γ_3, but the values are much smaller. These facts must be taken into account when comparing γ's obtained from various data sets. They may be also important for understanding the origin of the asphericity reflected in p-mode frequencies.

In Fig. 3, we present mean values of the γ's from various sources which includes BBSO (Libbrecht and Woodard, private communication), LOWL (Tomczyk, private communication) and SOHO/MDI (SOHO

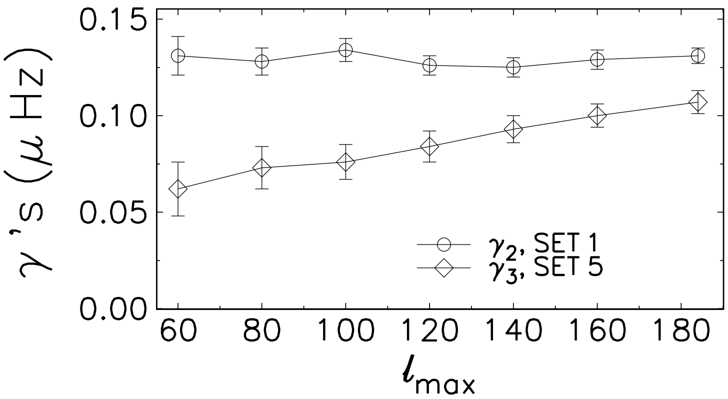

Fig. 2. Values of γ_2 and γ_3 inferred from data sets truncated at specified values of ℓ. Effect of truncation is similar for the other three sets.

team, private communication). We compare them with the monthly averages of smoothed sunspot numbers (NSO website). Clearly, the BBSO of 1988 and 1989 give the largest magnitudes of the γ's and this corresponds to the first half of the previous sunspot maximum. Unfortunately, we are missing splitting data from the rising phase (1987 or so), as well as from the declining phase.

The errors in the BBSO and LOWL γ's are about twice as large as those in SOHO/MDI, but they are still relatively small and the errors would not be visible in Fig. 3. The relatively small errors in the low-order γ's encourages a search for the ν dependence.

The similarities in the values of γ_1 and γ_2 between BBSO86 and LOWL data has already been observed. We note that the sets are from last two activity minima. The values of γ_2 derived from LOWL data were divided into four quarters and showed a barely significant decreasing trend from the value of 0.22 to 0.20 mHz. The mean value from SET 1 measurements taken about one year and half later was 0.13 mHz. It decreased during next three-quarters of a year to 0.08 mHz, as seen in Fig. 1.

The most clear evidence of the onset of the rising phase of activity during this three quarter of a year period is the systematic increase of γ_3 from SOHO/MDI seen both in Figs. 1 and 3. In the lowest boxes of Fig. 3, we see that the values in the data fill the gap in γ_3 between BBSO86 and BBSO88. Still, even SET5 is closer to BBSO86 than to BBSO88, but the increase in γ_3 is evident.

In γ_1, we also observe an evolution toward the values it has in the Sun's more active state. This evolution presents the weakest case. One should note, however, that temporal changes in γ_1 were also lagging behind those in the higher order γ's.

4. The near surface asphericity

The relatively small errors in the low-order γ's encourages a search for the ν dependence. The $\gamma_k(\nu)$ functions were determined by means of a least-square fit to the data assuming a three-term Legendre polynomial dependence.

In Fig. 4, we compare the $\gamma_k(\nu)$ inferred from SETs 1,3, and 5 of the SOHO/MDI data with those inferred by Dziembowski and Goode (1996) from the BBSO data of Libbrecht and Woodard and by Dziembowski et al. (1997) from the LOWL data of Tomczyk which he combined to a single set. One can see that the $\gamma(\nu)$ function determined from LOWL and SOHO data partially fill the gap between the BBSO datasets from 1986 and 1988.

The form of the $\gamma(\nu)$ function yields information about the sources of the solar distortion reflected in the even-a coefficients. The most direct inference is about localization. The relatively weak dependence tells us that the the perturbations resides within a few megameters of the photosphere.

Asphericities, as expressed by the γ's, imply that there is a non-radial perturbation of the Sun's mechanical equilibrium. The connection to magnetic fields is obvious in times of high activity, but we should also consider the possible role of the Reynold's stress. The stress can arise from rotation, meridional circulation and convective velocities. The rotational shape distortion is essentially time-independent, and so, we can quantify and remove it. The effect of meridional circulation is most likely

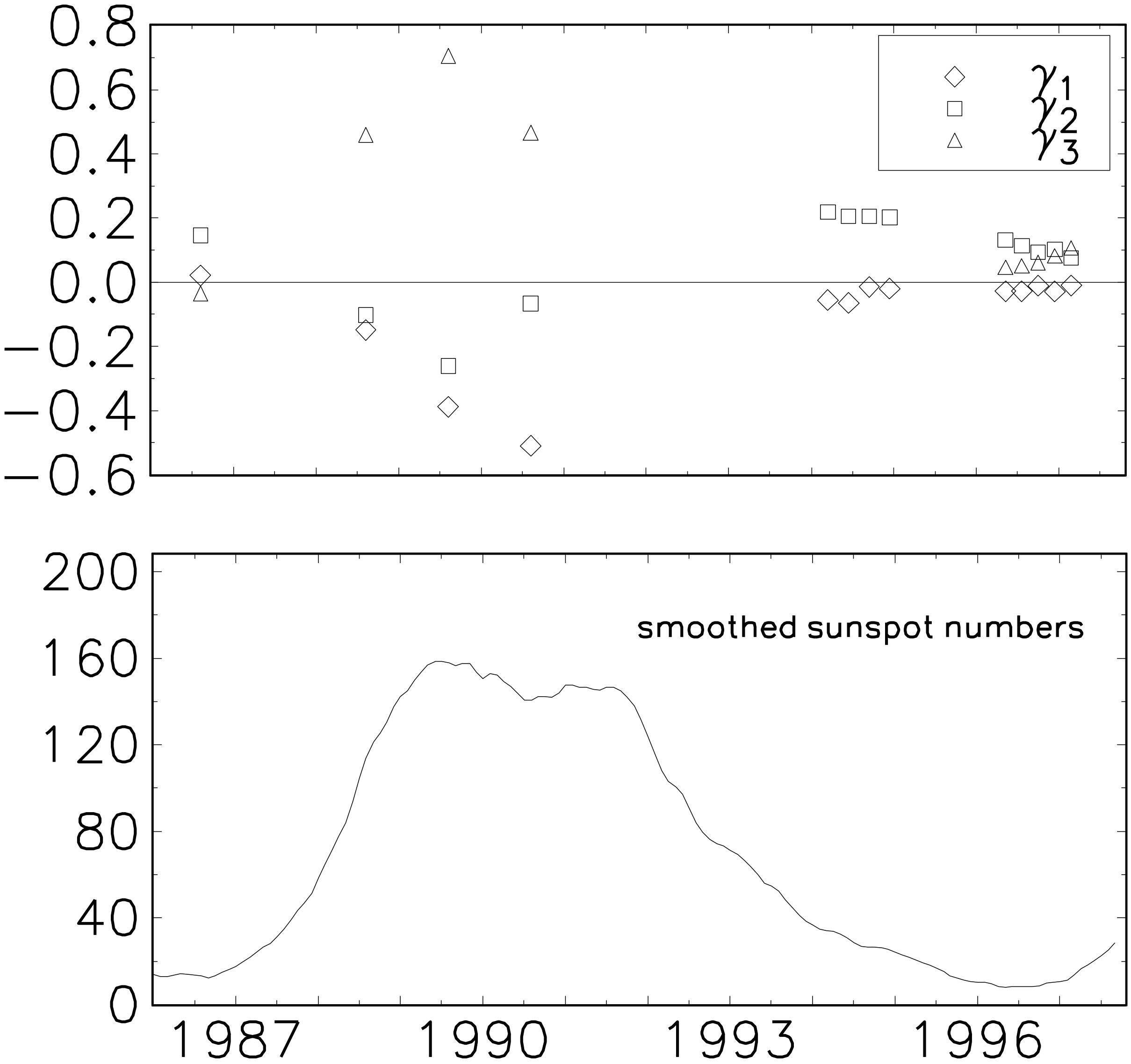

Fig. 3. The lower panel is a smoothed monthly average of the sunspot number covering the time since the 1986 activity minimum. The upper panel is a combination of γ's derived from BBSO (1986-1990), LOWL (1994) and SOHO/MDI (1996-) data. The errors in the γ's are smaller than the symbols used to represent their values.

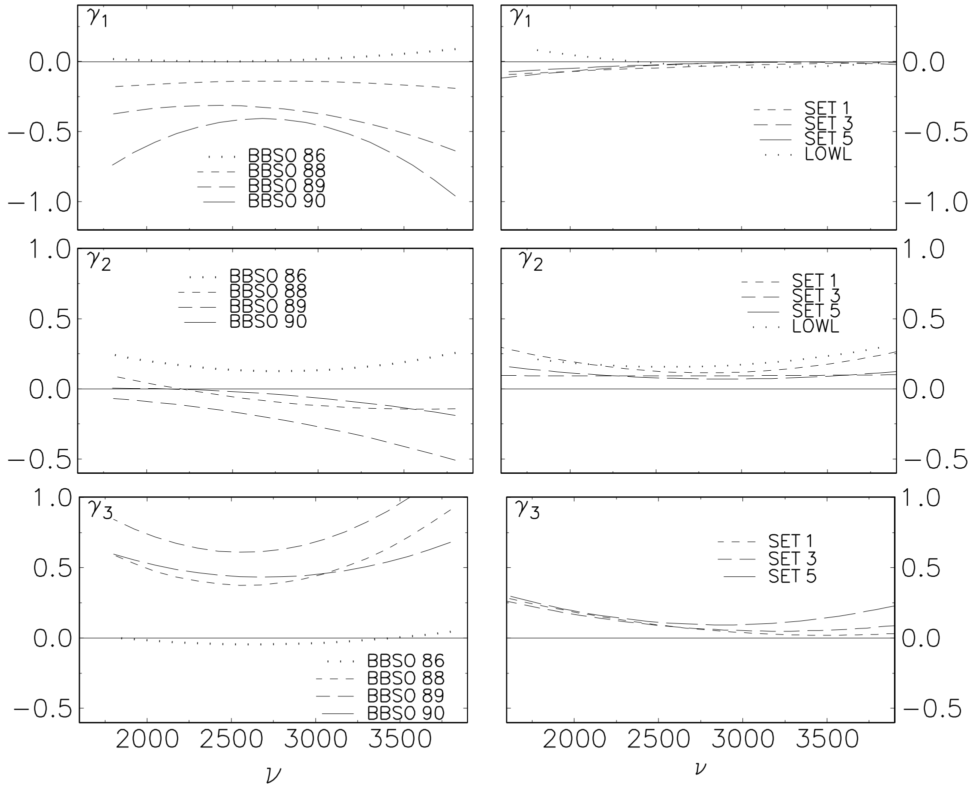

Fig. 4. The $\gamma_k(\nu)$ dependence from various data sets. The BBSO observations were made in the summers of the indicated years. The LOWL data were taken from February '94 to February '95. Measurements for SET 1, SET 3 and SET 5 were done in the intervals 05/01/96 – 07/11/96, 09/22/96–12/02/96, and 02/13/97–04/25/97, respectively.

negligible, as can be estimated from the recent measurement of its velocity by Giles, et al (1998). However, the convective flows can be both non-symmetric and time dependent. For instance, the "thermal shadow" of a deeply buried magnetic field could be reflected in global asymmetries in the near-surface velocity field. The Reynold's stress from convection is the only nonradial force implicitly included in the simulations of Kuhn and Stein (1996) which succesfully accounted for the sound speed asphericities detected with time-distance seismology by Duvall et al. (1996).

Thus, even though the cause of the asymmetries may be deeply buried, the source of the γ's are large scale structures near the surface. The most likely candidates are near surface convection (where convective velocities are the largest) or magnetic fields (with magnetic pressure being the closest to the gas pressure near the surface).

5. Asphericity in the interior

It would be of great interest to discover asymmetries seated in the deep interior. Unfortunately, a preliminary analysis of the SOHO/MDI data gives us little reason to hope. The residuals of the $a_{2k,\ell,n,\mathrm{res}}$, after removing the centrifugal term and the γ term, do not exhibit a visible dependence on the position of the inner turning point. It is well-known that p-modes preferentially sample the region just above their inner turning points.

In Figure 4, we plot the quantities

$$\gamma_{k,\mathrm{res}} = \frac{a_{2k,\mathrm{res}} C_k}{I}, \tag{5}$$

for $k = 2$ and 3 for SETs 1 & 5 (we dropped here ℓ and n subscripts) against the parameter λ which determines the position of the lower turning point. We see no trend in the distribution of the residuals about the zero-line, except, perhaps, some preponderance of positive values above $\lambda = 1.8$ which corresponds values of the fractional radius above 0.95. The region of special interest is the vicinity of the bottom of the convection zone at $r \approx 0.7R_\odot$, which corresponds to $\lambda \approx 1.05$. We do not see any feature indicating the P_4 or P_6 distortion in this region. The same is observed in corresponding plots for the remaining sets. There is no apparent evidence for a P_2 distortion, other than the centrifugal one, in the interior from $\gamma_{1,\mathrm{res}}$ plots.

NOTE: PLACE FOR LIMIT ON TOROIDAL FIELD?

6. Solar radius from the f-modes

Schou, et al. (1997) showed that the f-mode ridge in the SOHO/MDI k-ω diagram is so precisely determined that a constant offset between the true frequencies and their model counterparts became apparent. They noted that since the asymptotic f-mode frequency is $\sqrt{g\mathrm{k}}$, the only source of the offset could be an incorrect model radius. They determined that the seismic radius is about 300 km small than the model radius. They emphasized that the f-mode data open the possibility of determining the cycle dependence of the solar radius.

We have already seen small variations in the seismic radius with the five SOHO/MDI data sets we have. Each of the sets contains data for 33 f-modes. We only made use of their centroid frequencies. Separately, for each set we determined corrections to the standard solar model solar radius. The corrections were evaluated from

$$\frac{\Delta R}{R_\odot} = -1.5 < \frac{\Delta \nu_{\ell,0}}{\nu_{\ell,0}} >, \tag{6}$$

where <> denotes the average weighted by the inverse square of the measurement errors. We used standard solar model of Christensen-Dalsgaard et al.(1996) The results are given in Fig. 6. The differences among the sets are bigger than the formal errors (which are very small indeed). However, only subsequent data will tell us about the reality of the apparent radius minimum from SET3 which coincides with the minimum of the sun's activity (see Fig. 3).

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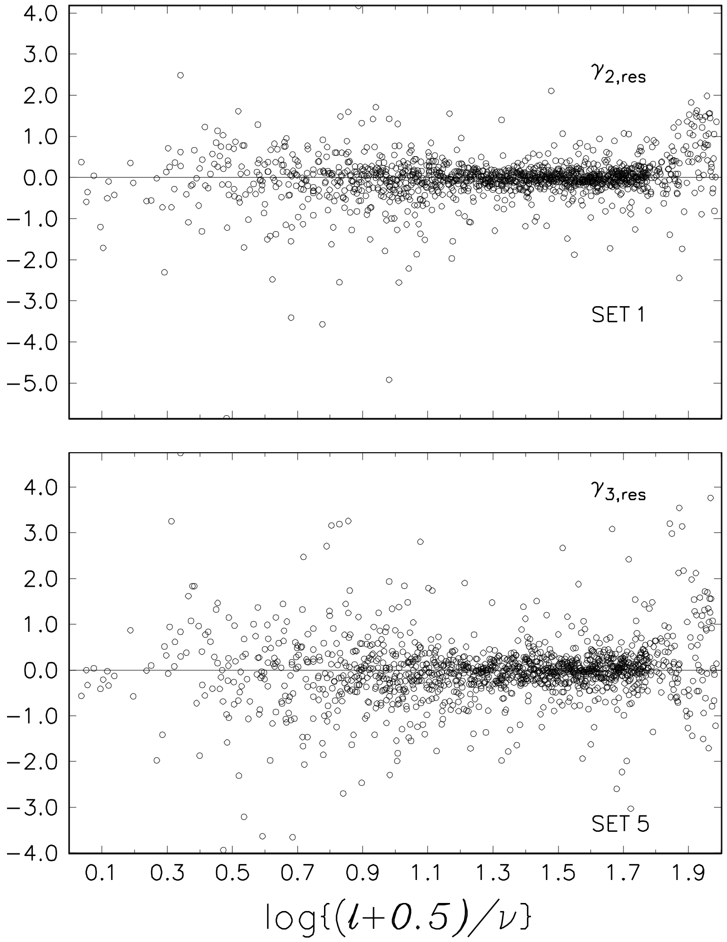

Fig. 5 The residual $\gamma_{k,\mathrm{res}}$ (see Eq.3) for $k = 2$ and 3 for SETs 1& 5

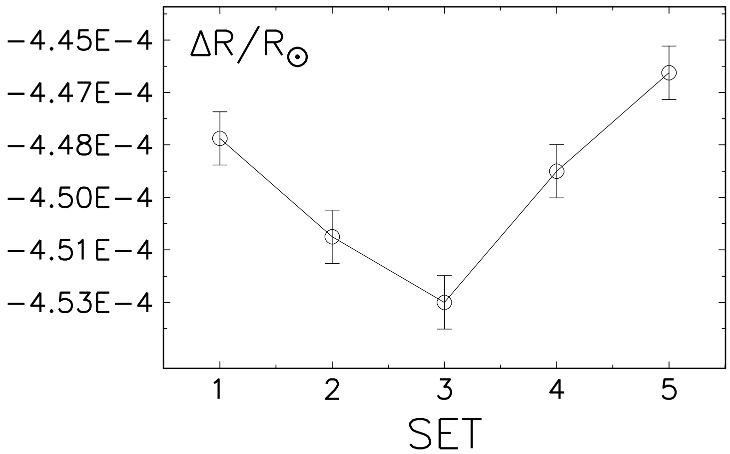

Fig. 6 Relative differences in radius of the sun inferred from the f-mode frequencies and the standard solar model.

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