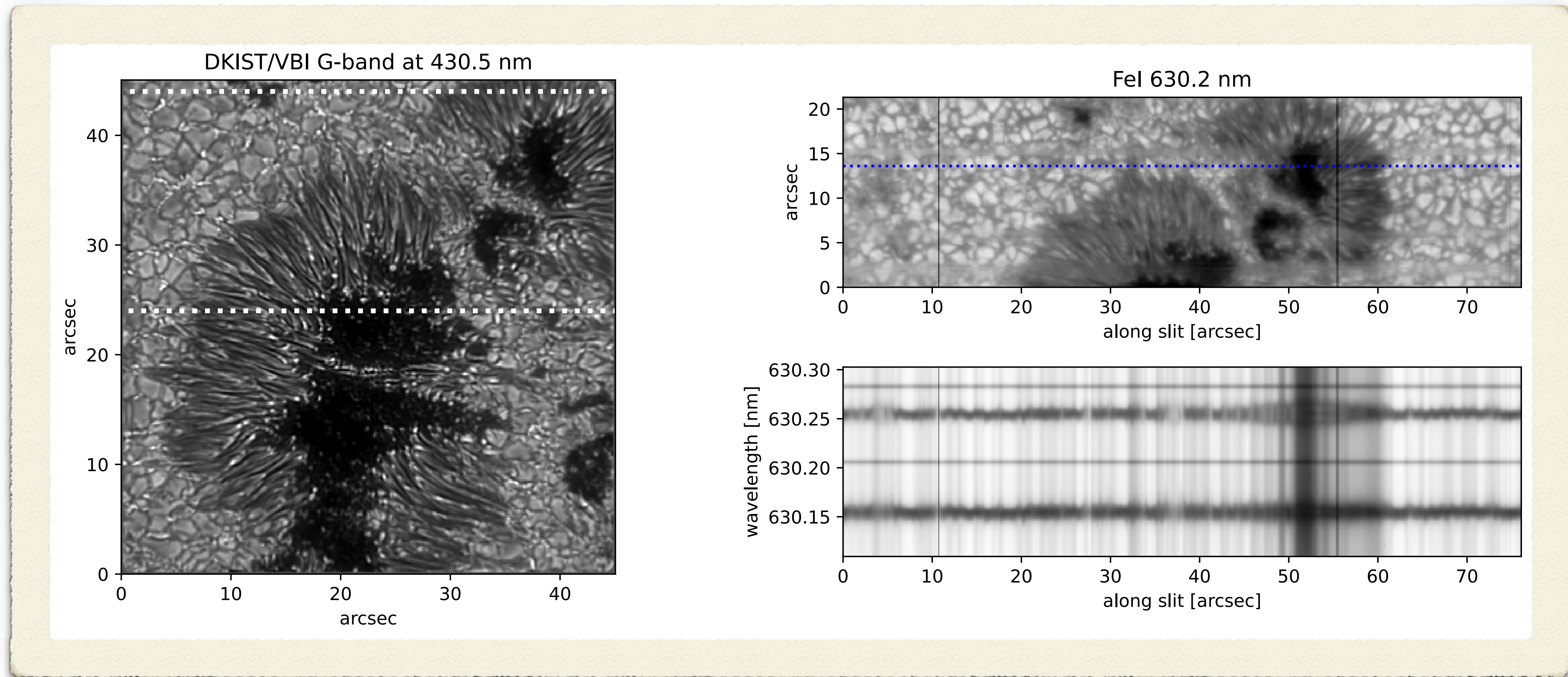


# High-resolution solar spectropolarimetry and magnetic field diagnostics

David Kuridze



# Outline

- Description of polarized radiation
  - Stokes formalizm
- Measurements of the Stokes parameters
- The Zeeman effect

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- Description of polarized radiation
  - Stokes formalism
- Measurements of the Stokes parameters
- The Zeeman effect
- Polarization observations with DKIST
  - Measurements/calibrations
- Inferring the magnetic field
  - Polarized Radiative Transfer
  - Inversion, Weak field approximation, Center of Gravity

# Description of Polarized radiation

## Polarization ellipse

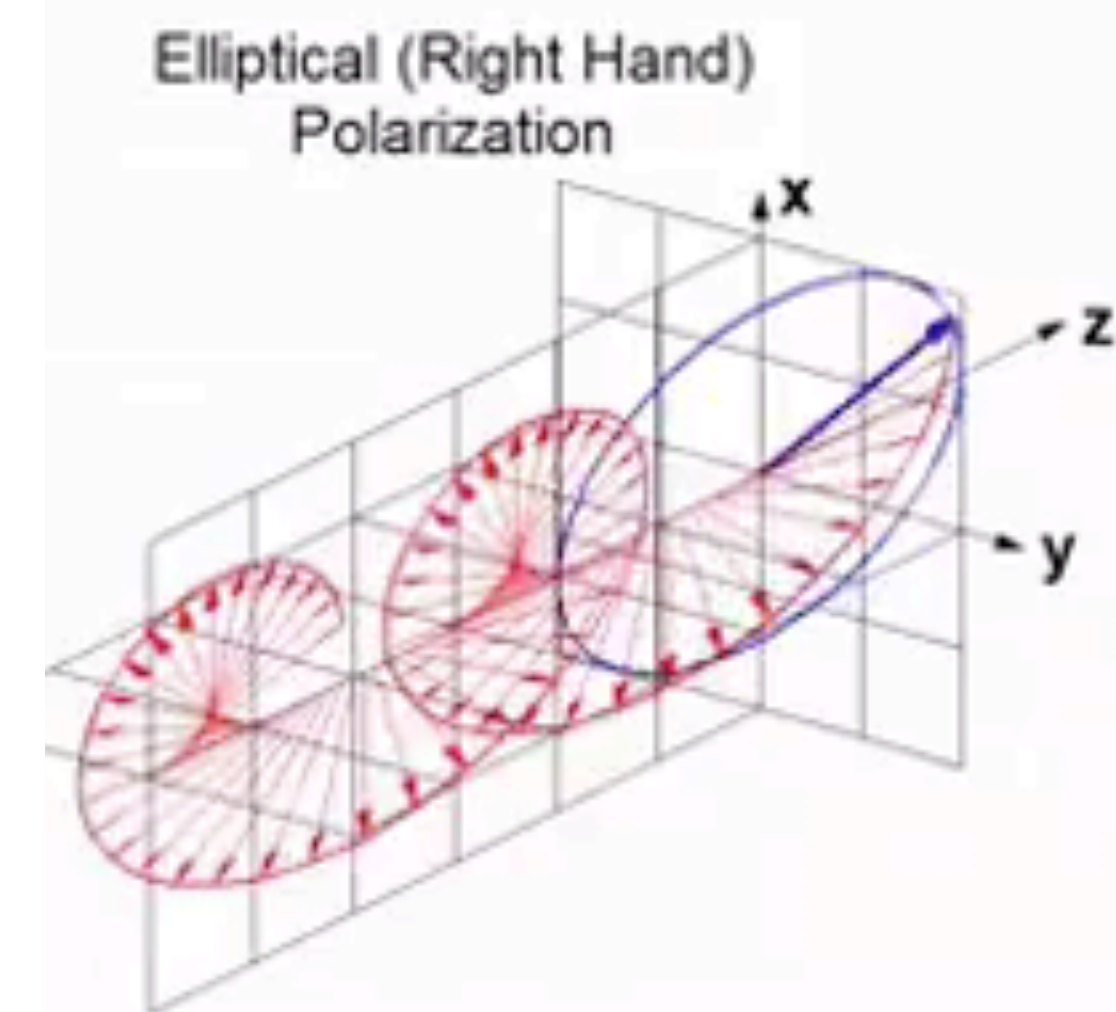
- Electric and magnetic field in the radiation beam are perpendicular to the direction of propagation

- $E_x(\vec{r}, t) = A_x e^{-i(\omega t + \phi_x)}$

- $E_y(\vec{r}, t) = A_y e^{-i(\omega t - \phi_y)}$

- $\phi = \phi_x - \phi_y$

- $\frac{E_x^2}{A_x^2} + \frac{E_y^2}{A_y^2} + 2\frac{E_x E_y}{A_x A_y} \cos \phi = \sin^2 \phi$



# Description of Polarized radiation

## Polarization ellipse

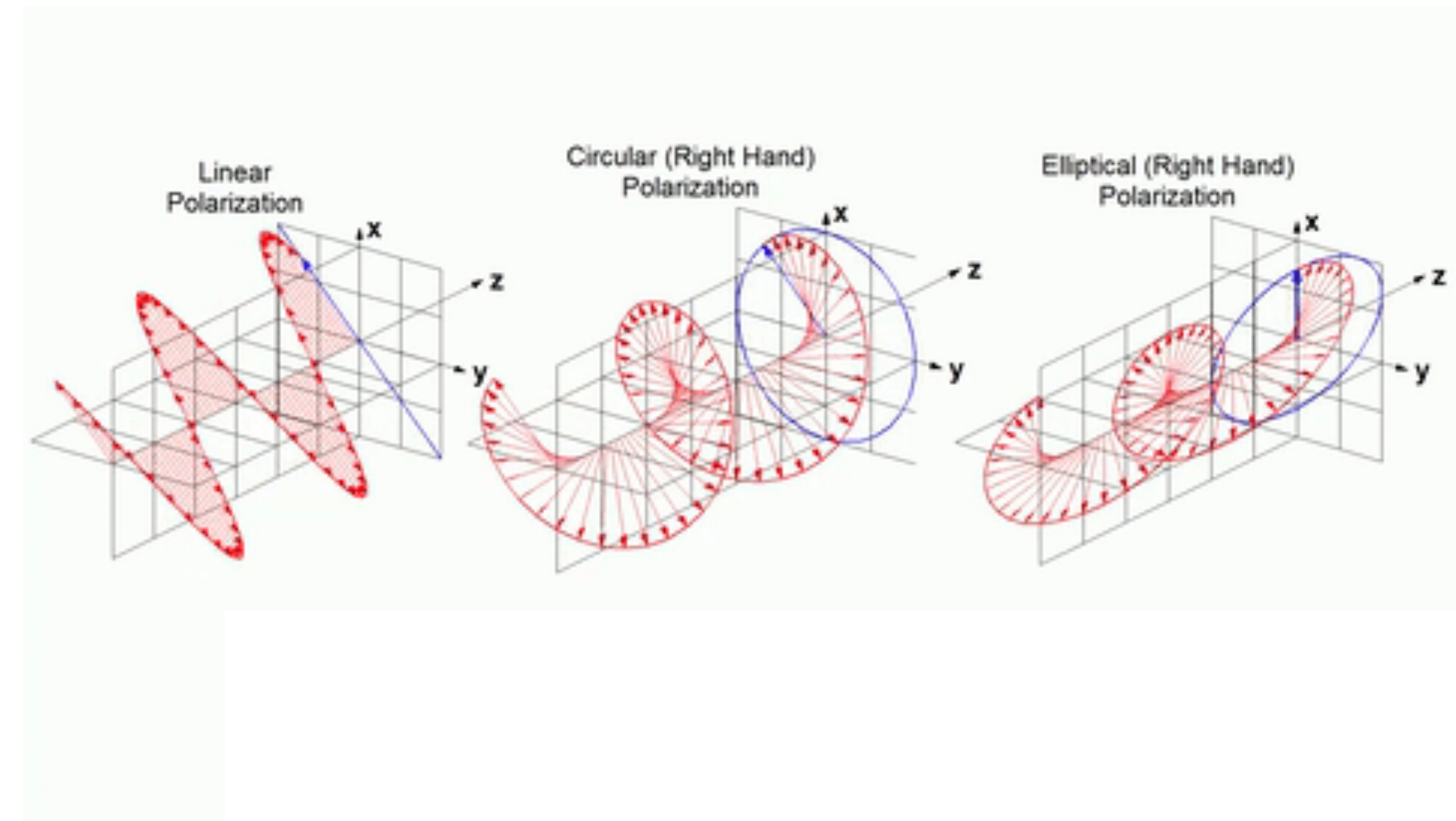
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# Description of Polarized radiation

## Polarization tensor

- $E_x(\vec{r}, t) = A_x e^{-i(\omega t + \phi_x)}$
- $E_y(\vec{r}, t) = A_y e^{-i(\omega t + \phi_y)}$
- $I = \frac{c}{8\pi} \langle E_i^* E_j \rangle$ ,  $i, j = x, y$ .
- $I = \begin{pmatrix} \langle E_x^* E_y \rangle & \langle E_y^* E_x \rangle \\ \langle E_x^* E_x \rangle & \langle E_y^* E_y \rangle \end{pmatrix} = \begin{pmatrix} \langle A_x^2 \rangle & \langle A_x^* A_y e^{i\phi} \rangle \\ \langle A_x^* A_y e^{-i\phi} \rangle & \langle A_y^2 \rangle \end{pmatrix}$

# Description of Polarized radiation

## Polarization tensor and Stokes parameters

$$\bullet I = \begin{pmatrix} \langle E_x^* E_y \rangle & \langle E_y^* E_x \rangle \\ \langle E_x^* E_y \rangle & \langle E_y^* E_y \rangle \end{pmatrix} = \begin{pmatrix} \langle A_x^2 \rangle & \langle A_x^* A_y e^{i\phi} \rangle \\ \langle A_x^* A_y e^{-i\phi} \rangle & \langle A_y^2 \rangle \end{pmatrix}$$

$$\bullet I = \langle A_x^2 \rangle + \langle A_y^2 \rangle$$

$$\bullet Q = \langle A_x^2 \rangle - \langle A_y^2 \rangle$$

$$\bullet U = \langle A_x A_y \cos(\phi) \rangle$$

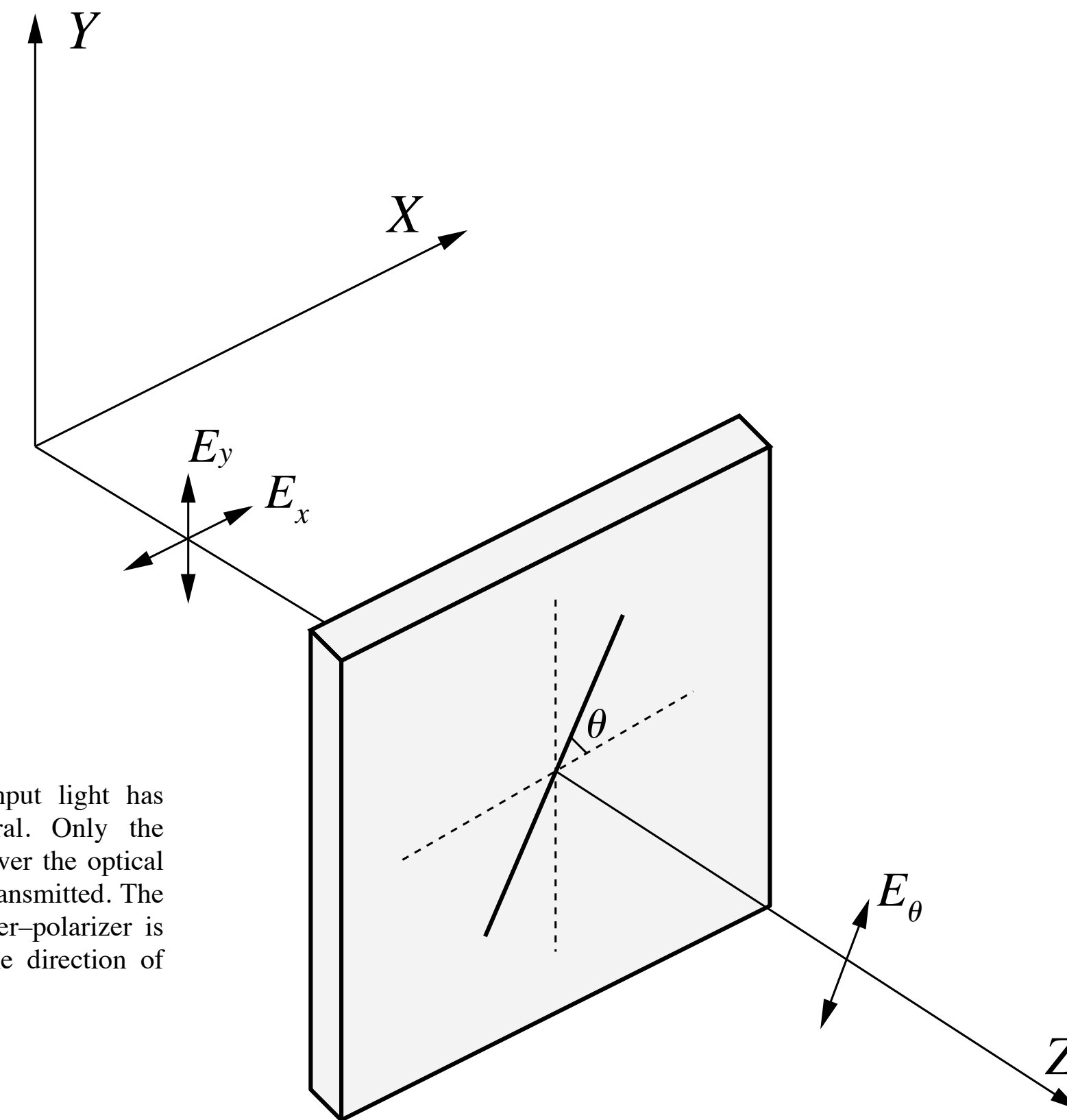
$$\bullet V = \langle A_x A_y \sin(\phi) \rangle$$

# Measurements of the Stokes parameters

- Linear polariser is a device that is totally transparent to the electric field along a given axes (called transmission axes) and totally opaque to the electric field along the axes perpendicular to the transmission axes

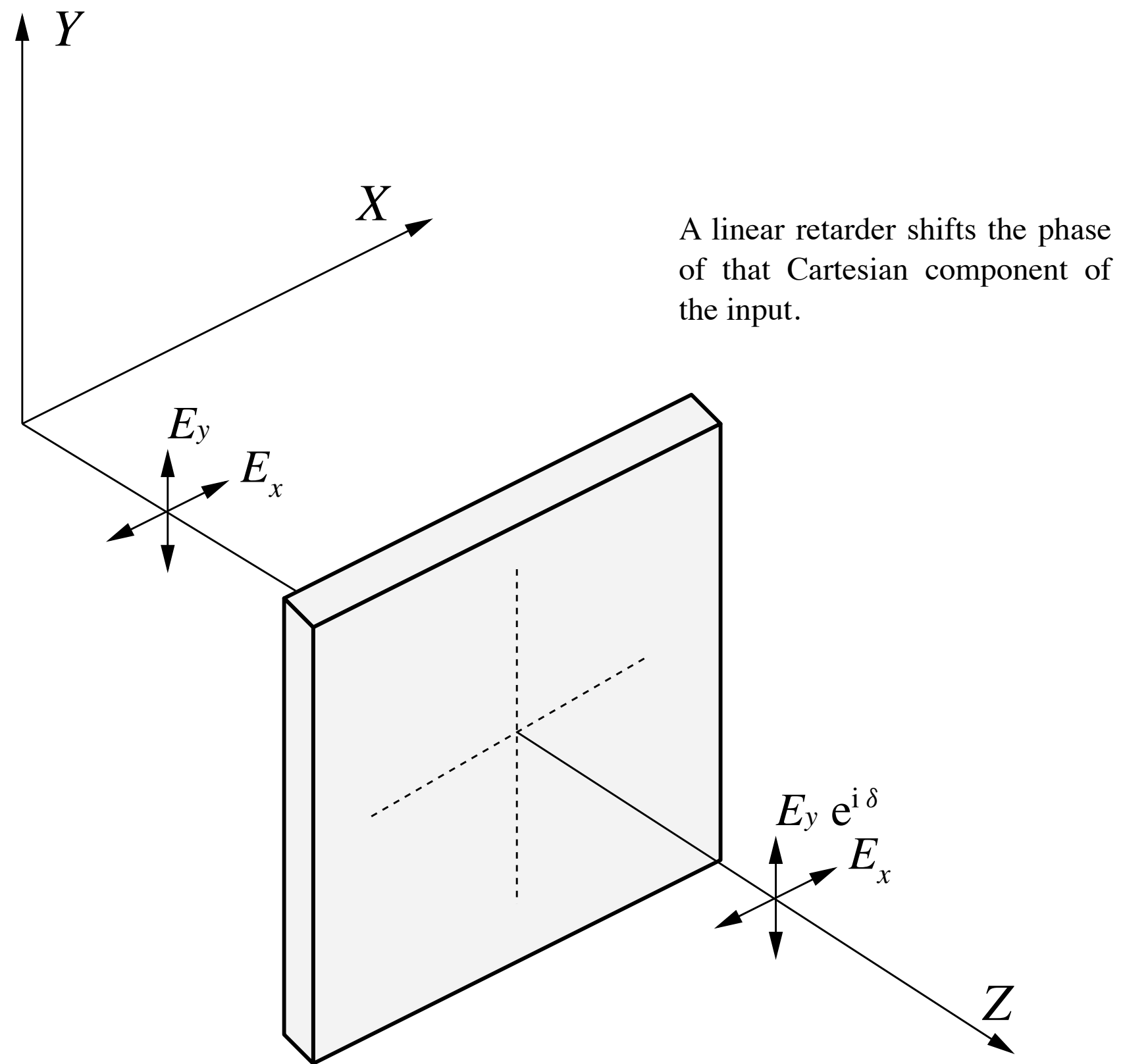
$$E'_x = E_x \cos \theta; \quad E'_y = E_y \sin \theta.$$

Linear analyzer-polarizer behavior. Input light has two Cartesian components in general. Only the projections of these two components over the optical axis (at an angle  $\theta$  to the  $X$  axis) are transmitted. The resulting beam from the linear analyzer-polarizer is completely linearly polarized along the direction of the optical axis.





# Measurements of the Stokes parameters



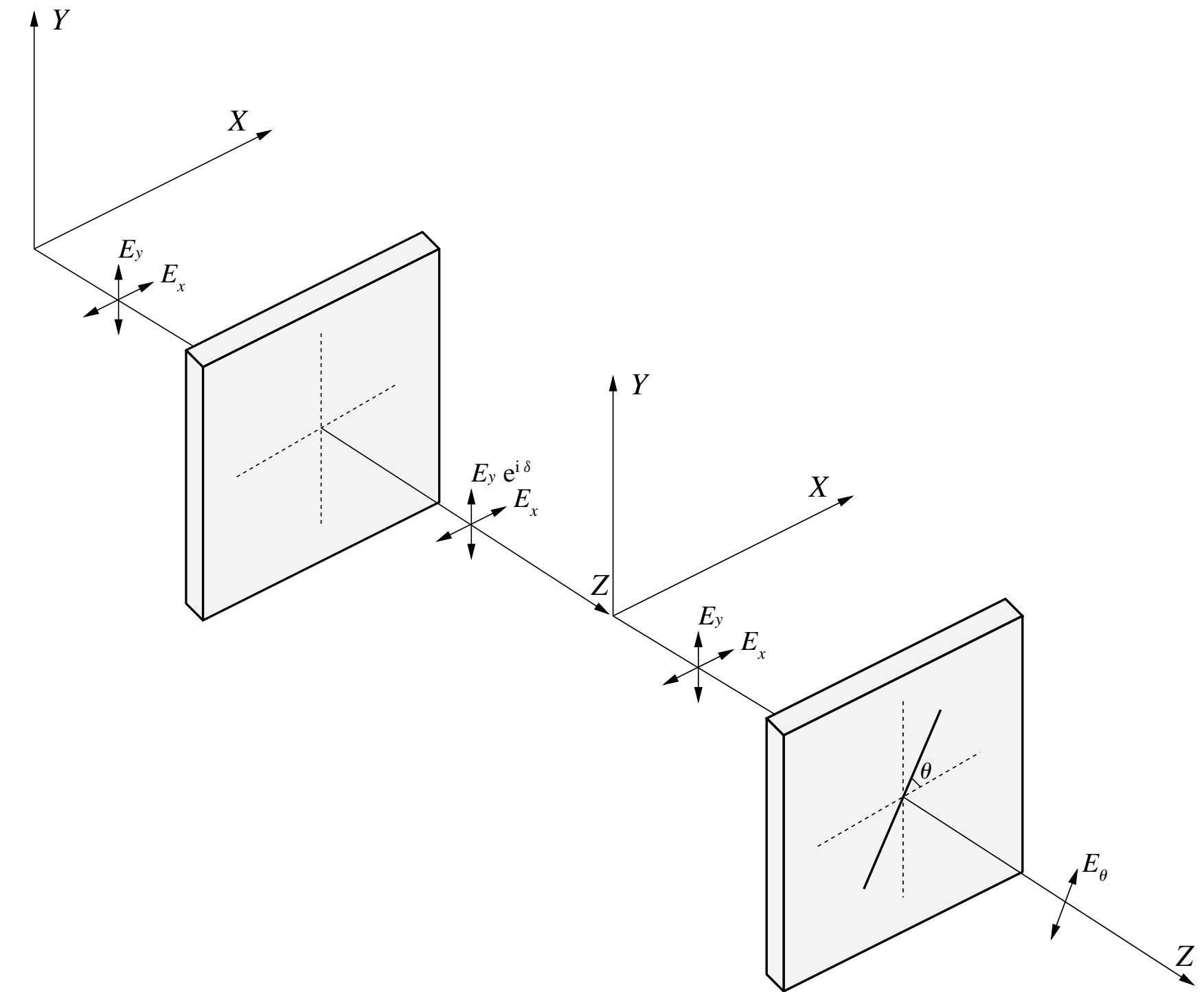
# Measurements of the Stokes parameters

$$E'_x = E_x \cos \theta; \quad E'_y = E_y \sin \theta.$$

$$E'_x = E'_x; \quad E'_y = E_y e^{i\delta}$$

*At the exit of the analyzer the light is completely linearly polarized at an angle  $\theta$ , the amplitude of the electric vector being given by*

$$E_\theta(\delta) = E_x \cos \theta + E_y \sin \theta e^{i\delta}$$



# Measurements of the Stokes parameters

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At the exit of the analyzer the light is completely linearly polarized at an angle  $\theta$ , the amplitude of the electric vector being given by

$$E_\theta(\delta) = E_x \cos \theta + E_y \sin \theta e^{i\delta}$$

The intensity of the output beam is

$$I_{meas}(\theta, \delta) = \langle E_\theta(\delta) E_\theta^*(\delta) \rangle$$

$$I_{meas}(\theta, \delta) = \langle E_x E_x^* \cos^2 \theta + E_y E_y^* \sin^2 \theta + \frac{1}{2} E_x E_y^* \sin 2\theta e^{-i\delta} + \frac{1}{2} E_x^* E_y \sin 2\theta e^{i\delta} \rangle$$

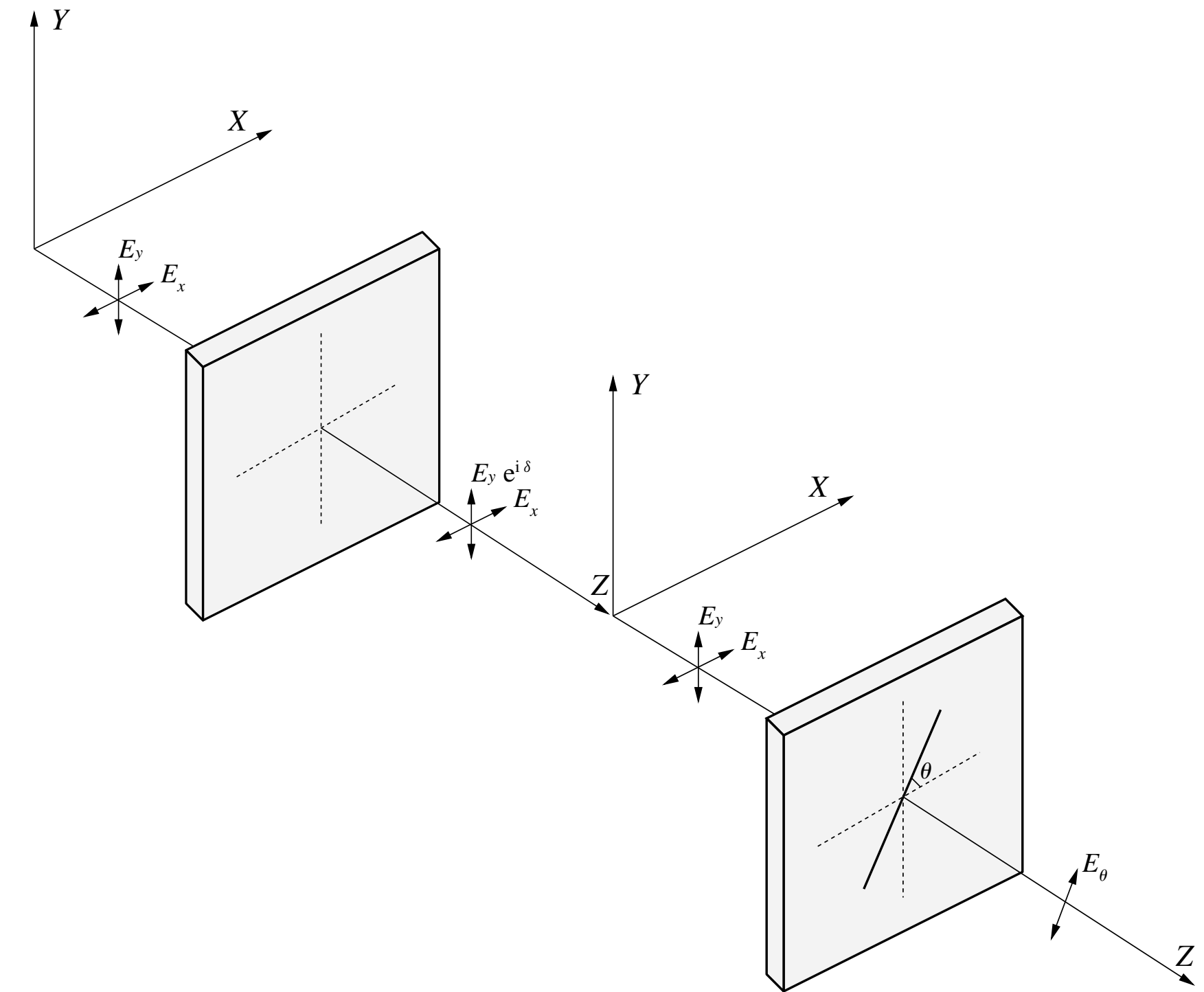
$$I = \langle E_{0x}^2 \rangle + \langle E_{0y}^2 \rangle$$

$$Q = \langle E_{0x}^2 \rangle - \langle E_{0y}^2 \rangle$$

$$U = \langle E_{0x} E_{0y} \cos \delta \rangle$$

$$V = \langle E_{0x} E_{0y} \sin \delta \rangle$$

$$I_{meas}(\theta, \delta) = \frac{1}{2} (I + Q \cos 2\theta + U \cos \delta \sin 2\theta + V \sin \delta \sin 2\theta)$$



# Measurements of the Stokes parameters

$$I_{meas}(\theta, \delta) = \frac{1}{2}(I + Q \cos 2\theta + U \cos \delta \sin 2\theta + V \sin \delta \sin 2\theta)$$

$$I = I_{meas}(0, 0) + I_{meas}(\pi/2, 0),$$

$$Q = I_{meas}(0, 0) - I_{meas}(\pi/2, 0),$$

$$U = I_{meas}(\pi/4, 0) - I_{meas}(3\pi/4, 0),$$

$$V = I_{meas}(\pi/4, \pi/2) - I_{meas}(3\pi/4, \pi/2).$$

$$I = \circlearrowleft + \circlearrowright \text{ or } \leftrightarrow + \updownarrow \text{ etc}$$

$$Q = \updownarrow - \leftrightarrow ,$$

$$U = \nearrow - \searrow ,$$

$$V = \circlearrowleft - \circlearrowright ,$$

- These are modulations
- We measure them to reconstruct Stokes I/Q/U/V

# Polarization of the light

- Light is polarised due to a anisotropic physical state in the light source or media through which the light propagates
  - Magnetic field - Zeeman affect
  - Electric field - Stark affect
  - Scattering polarisation
  - Scattering polarisation + magnetic field (Hanle affect)
  - Impact polarisation
  - Atomic polarisation

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  - Impact polarization
  - Atomic polarisation

# Muller Matrix

- Influence of any medium/material, physical element (telescope optic, mirrors, lens ...) on polarized radiation can be described through  $4 \times 4$  matrices, called Muller matrices
- Polarization calibrations are based on determination of telescopes' Muller matrix.

$$\begin{aligned} & \mathbf{I}' = \mathbf{M}\mathbf{I}, \\ \begin{pmatrix} I' \\ Q' \\ U' \\ V' \end{pmatrix} &= \begin{pmatrix} M_{00} & M_{01} & M_{02} & M_{03} \\ M_{10} & M_{11} & M_{12} & M_{13} \\ M_{20} & M_{21} & M_{22} & M_{23} \\ M_{30} & M_{31} & M_{32} & M_{33} \end{pmatrix} \begin{pmatrix} I \\ Q \\ U \\ V \end{pmatrix} \end{aligned}$$

# Atomic structure

**Each bound energy level is characterised by four quantum numbers:**

- Principal quantum number,  $n$
- Orbital angular momentum,  $l$  ( $l=0, 1, 2, 3, \dots, n-1$ ;  $s, p, d, f$ )
- Spin angular momentum,  $s$  ( $s=\pm 1/2$  for electrons,  $1, -1$  for photons)
- Total angular momentum,  $j=l+s$  ( $||l-s|\leq j\leq|l+s|$ )
- Total angular momentum for multi-electron atoms,  $J=L+S$  ( $L=\sum l_i$ ,  $S=\sum S_i$ ,  $|L-S|\leq J\leq|L+S|$ )
- Components of the total angular momentum ( $J$ ) in the direction of the external magnetic/electric field,  $M$  ( $2J+1$ )
- Atomic state is described as:  $|l s j m\rangle$

$$\mathbf{H}_0 |l s j m\rangle = E_j |l s j m\rangle$$



# Atomic structure

**Each bound energy level is characterised by four quantum numbers:**

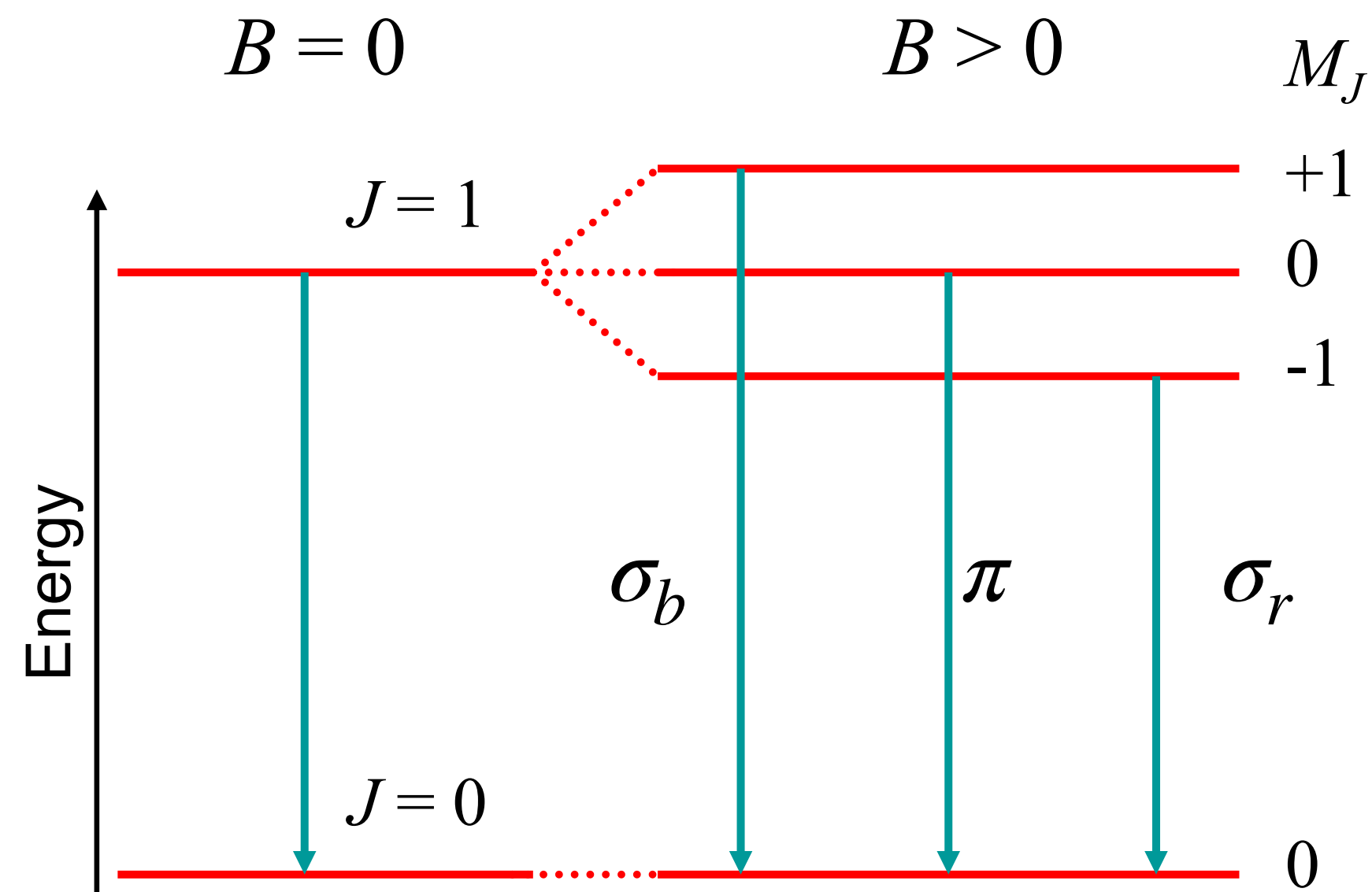
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- Components of the total angular momentum ( $J$ ) in the direction of the external magnetic/electric field,  $M$  ( $2J+1$ )
- Atomic state is described as:  $|lsjm\rangle$

$$\mathbf{H}_0 |lsjm\rangle = E_j |lsjm\rangle$$

$$(\mathbf{H}_0 + \mathbf{H}_B) |lsjm\rangle = (E_j + mg h\nu_L) |lsjm\rangle$$

# The Zeeman effect

- The presence of the magnetic field breaks degeneracy and an atomic level of total angular momentum  $J$  is split into  $(2J + 1)$  magnetic sublevels, where  $J = L + S$  ( $L$  is orbital angular momentum and  $S$  is spin angular momentum)



$$\Delta M = +1(\sigma_b), 0(\pi), -1(\sigma_b)$$

$$E = E_0 - (\boldsymbol{\mu} \cdot \mathbf{B}), \quad \boldsymbol{\mu} = \frac{1}{2} \left( \frac{e}{m} \frac{h}{2\pi} g M \right)$$

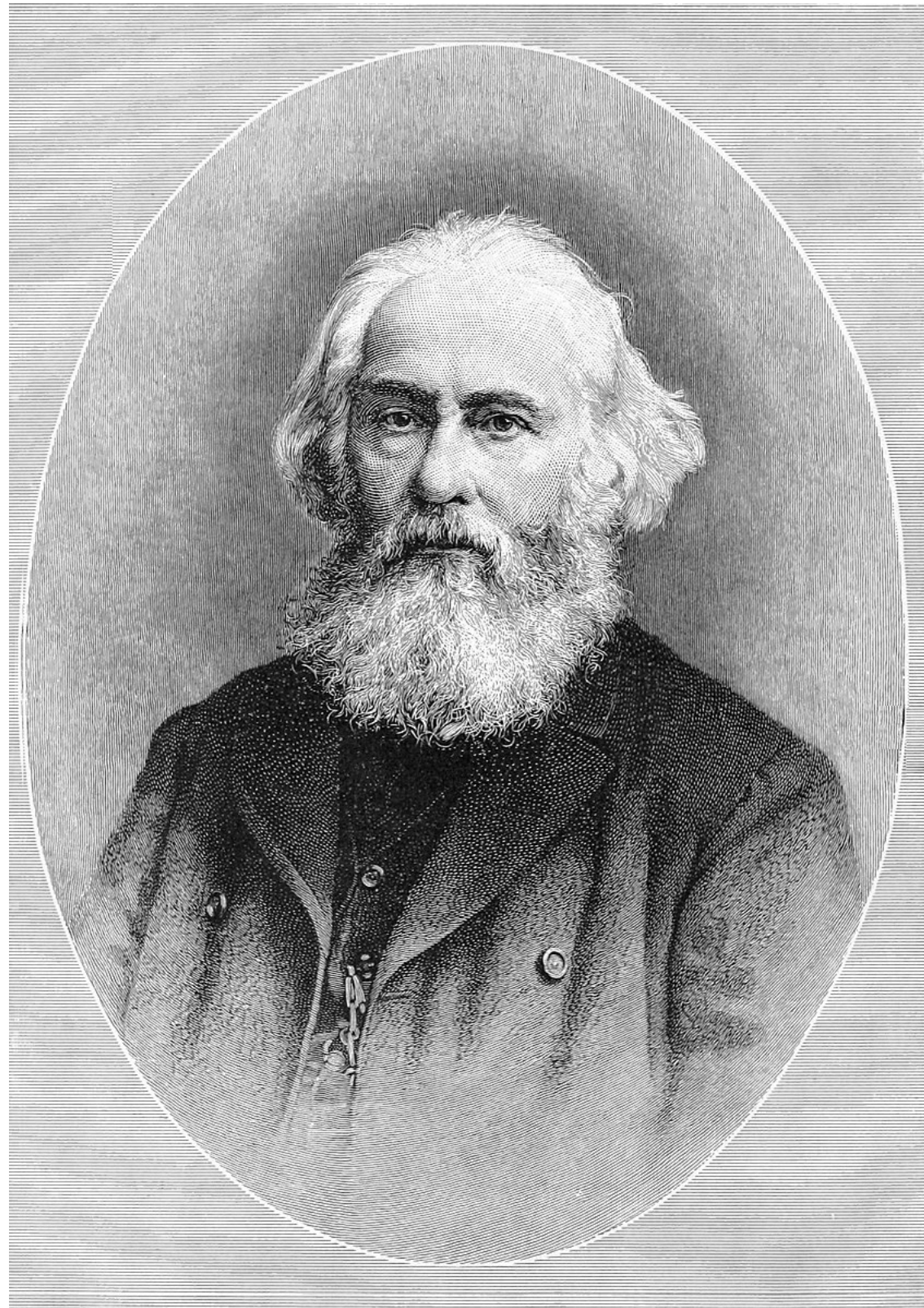
$$g = 1 + \frac{J(J+1) + S(S+1) - L(L+1)}{2J(J+1)}$$

$$\lambda - \lambda_0 = \frac{e}{4\pi m_e c} g^* \lambda^2 B$$

$$\Delta\lambda_H = 4.67 \times 10^{-13} g_{eff} \lambda_0^2 B_{LOS}$$

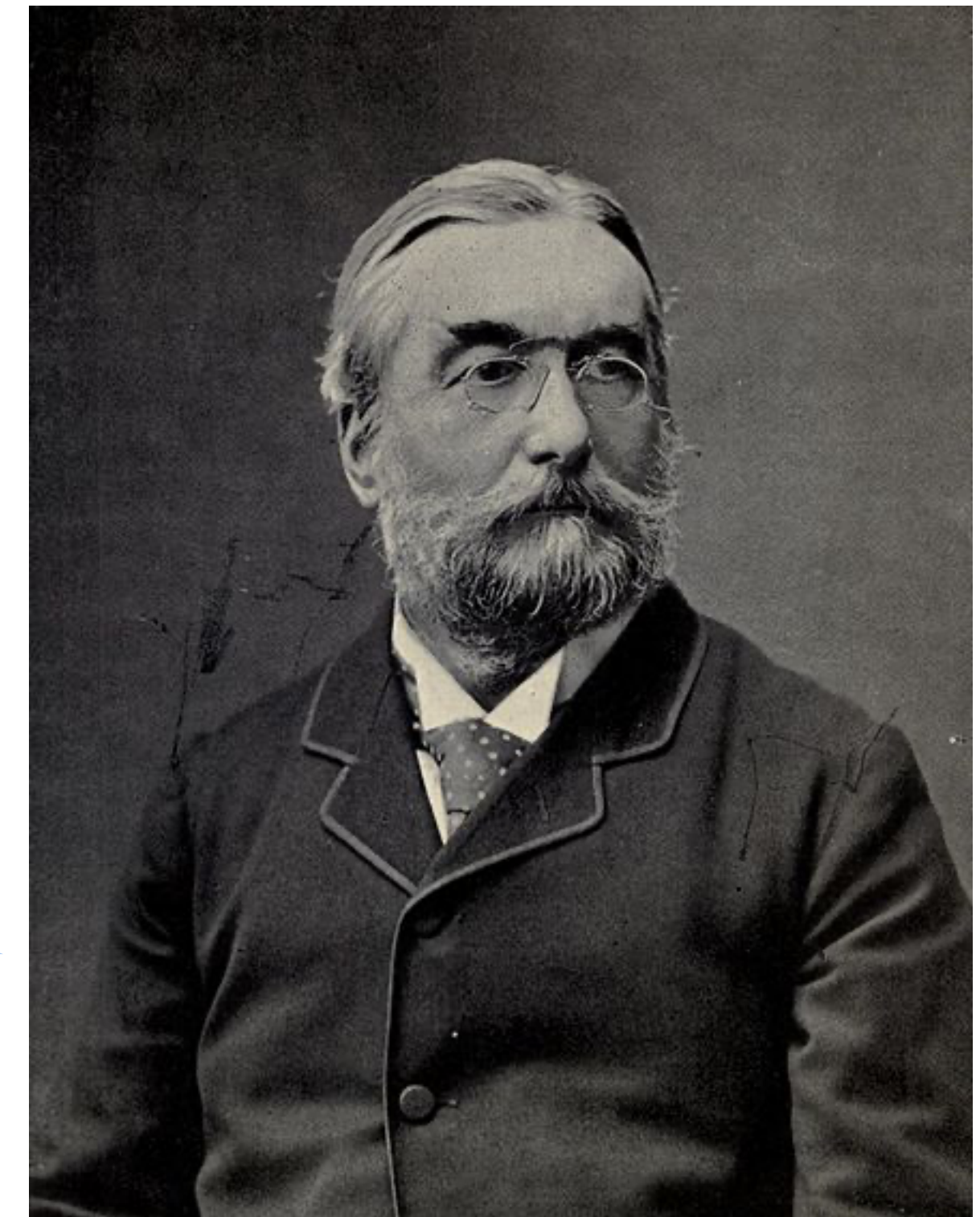
# First extraterrestrial magnetic field

William Huggins (1824 – 1910)



**The lines of the FRAUNHOFER appeared to be stronger and thicker in the spectrum of the umbra (1868)**

Norman Lockyer (1836 – 1920)

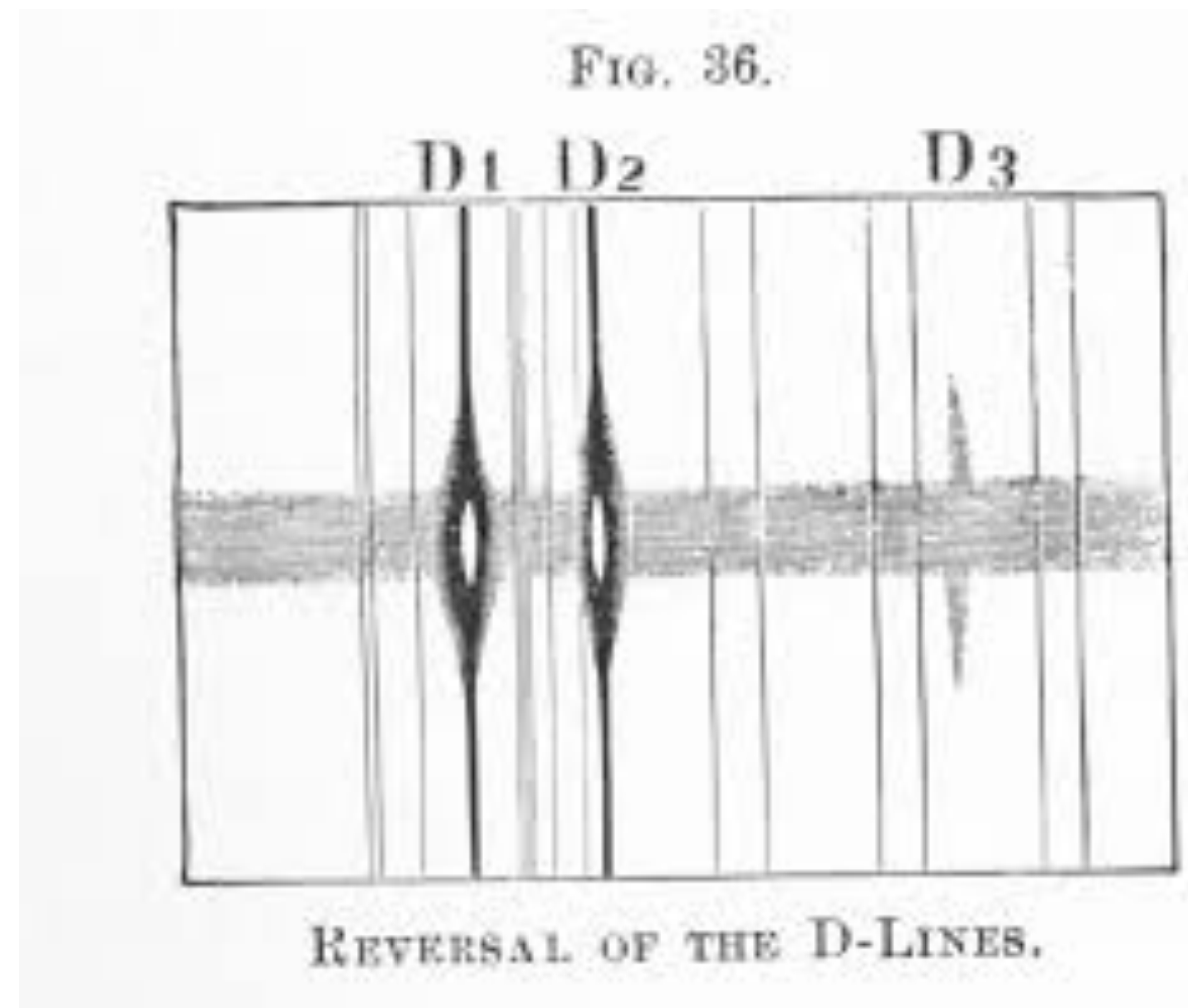


**All the absorption-bands, were visible in the spectrum of the spot; they moreover appeared thicker where they crossed the spot spectrum (1867) \*.**

\* Lockyer, J. Norman. 1867, Spectroscopic Observations of the Sun. Astronomical register, vol. 5, pp.38-39

# The Zeeman effect

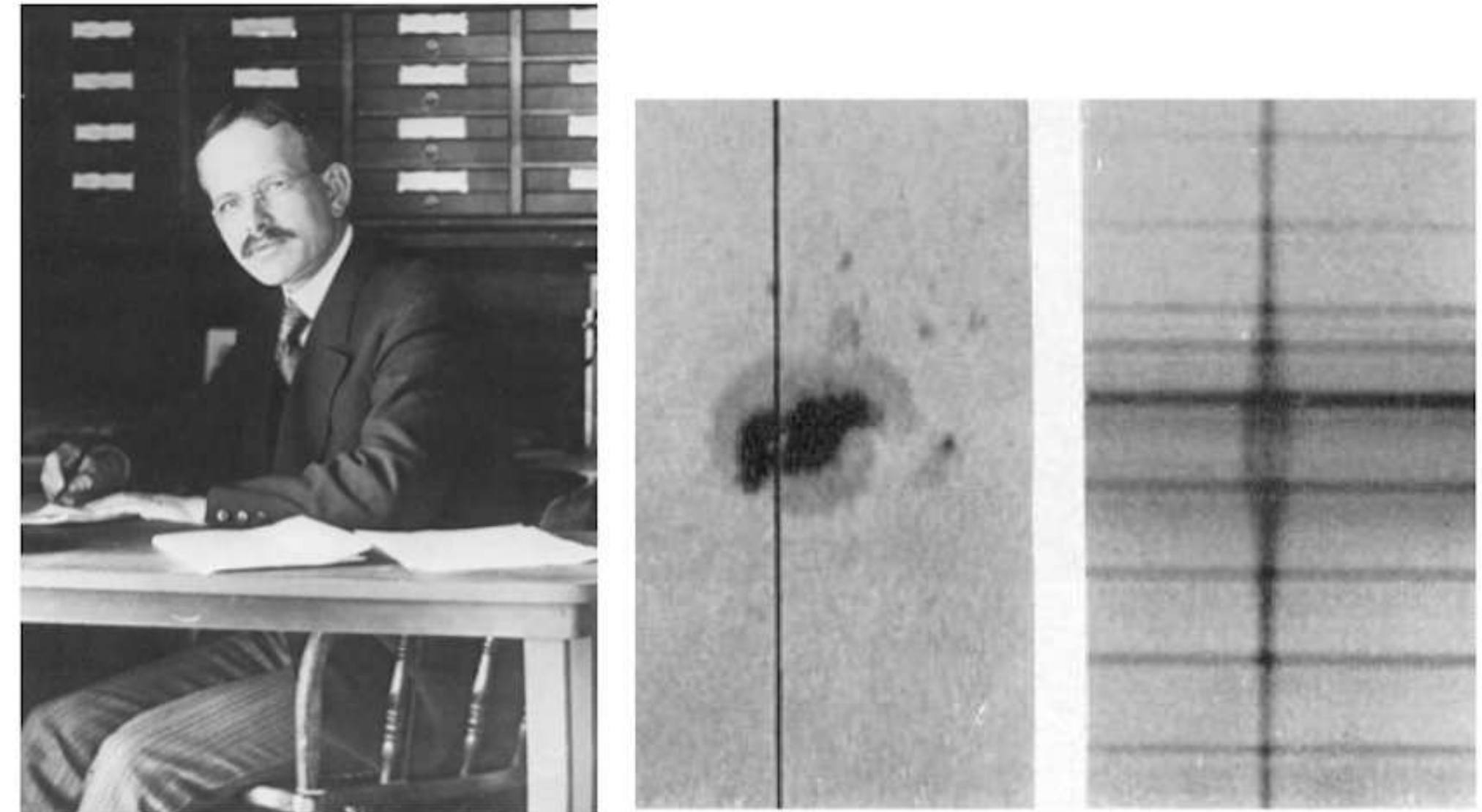
- Line splitting observed in 1870
- Zeeman effect discovered in 1896



Broadening and reversal of Na I D<sub>1</sub>  $\lambda$  5889.950 Å and Na I D<sub>2</sub>  $\lambda$  5895.924 Å

# The Zeeman effect

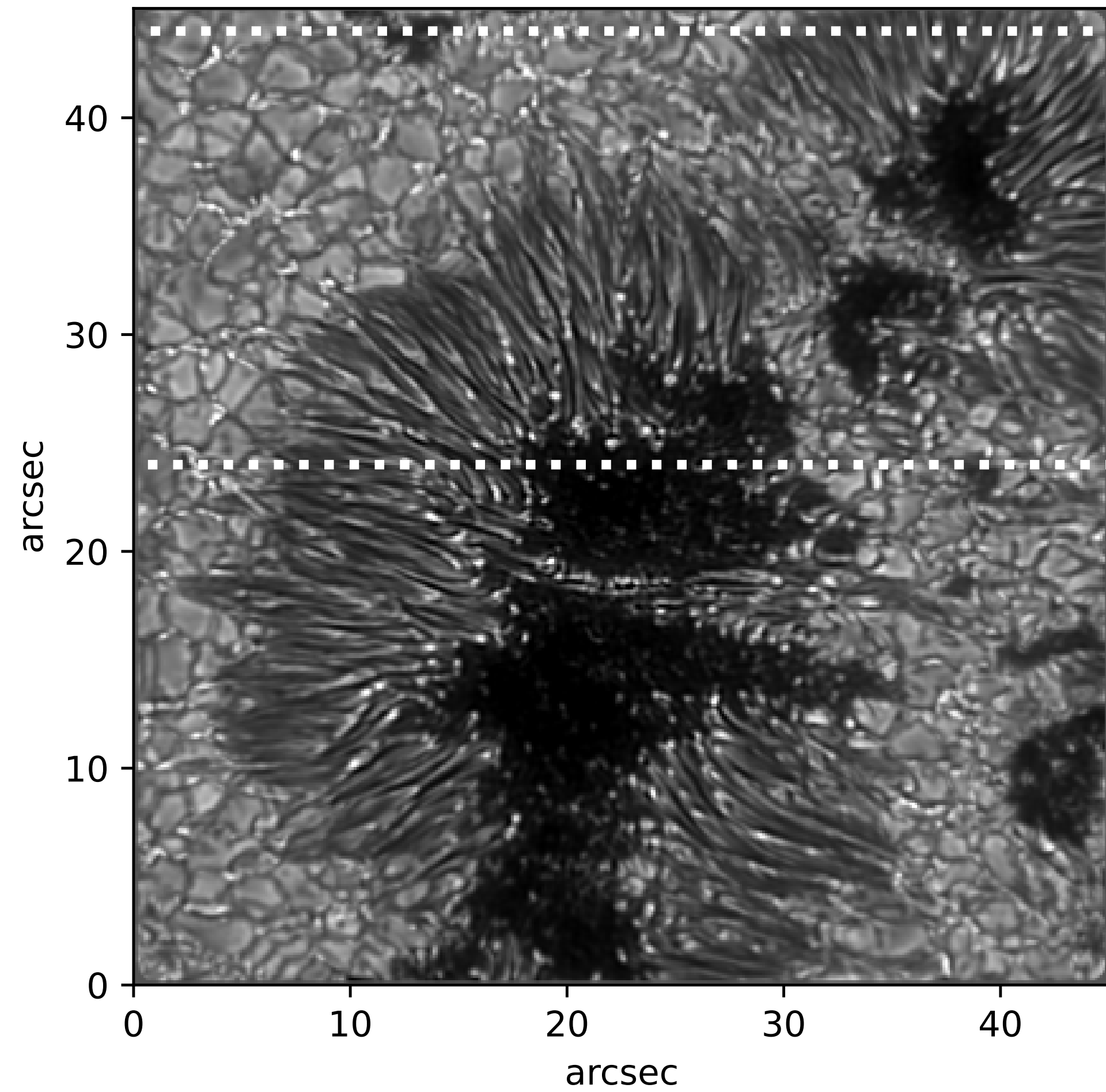
- Line splitting observed in 1870
- Zeeman effect discovered in 1896
- Discovery of the magnetic field in sunspots in 1908 (3000 G in sunspot umbrae) (*Hale 1908*)
  - *G.E. Hale, F. Ellerman, S.B. Nicholson, A.H. Joy, The Magnetic Polarity of Sun-Spots. Astrophys. J. 49, 153–178 (1919)*



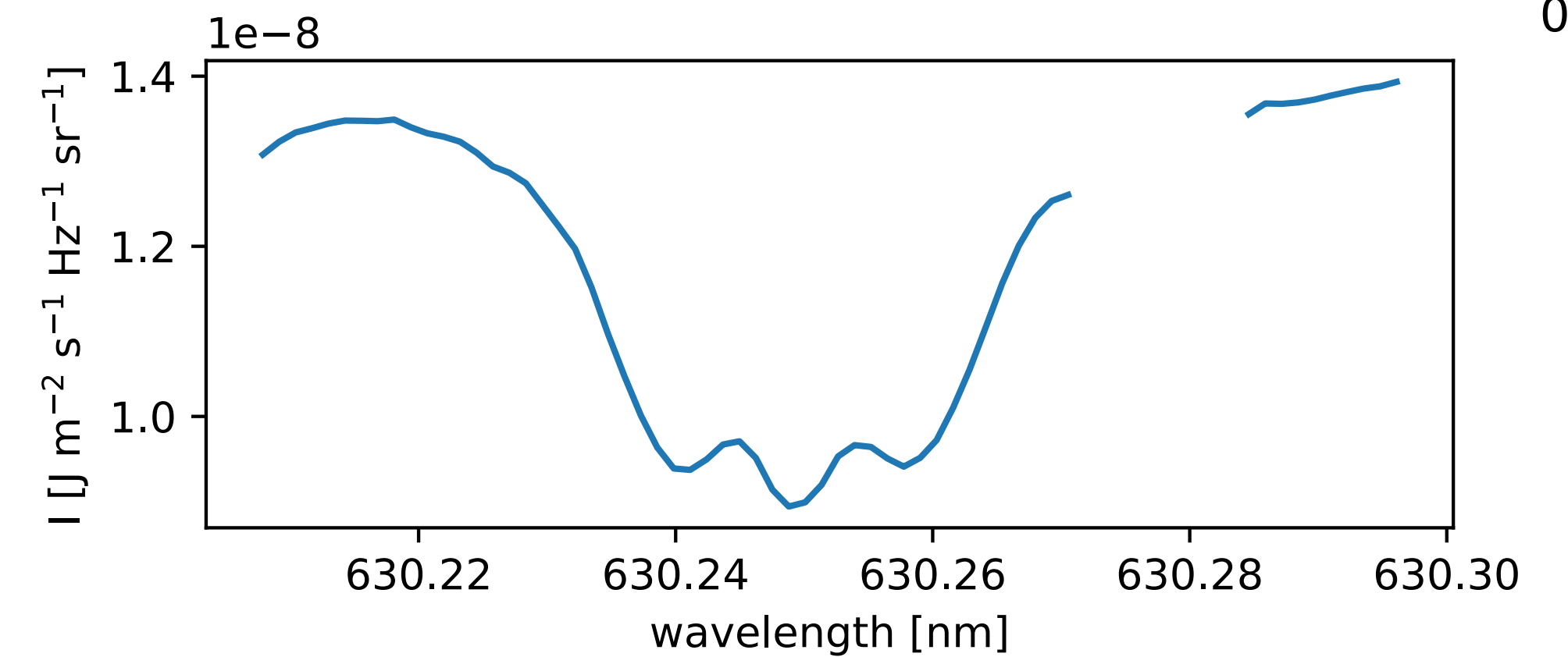
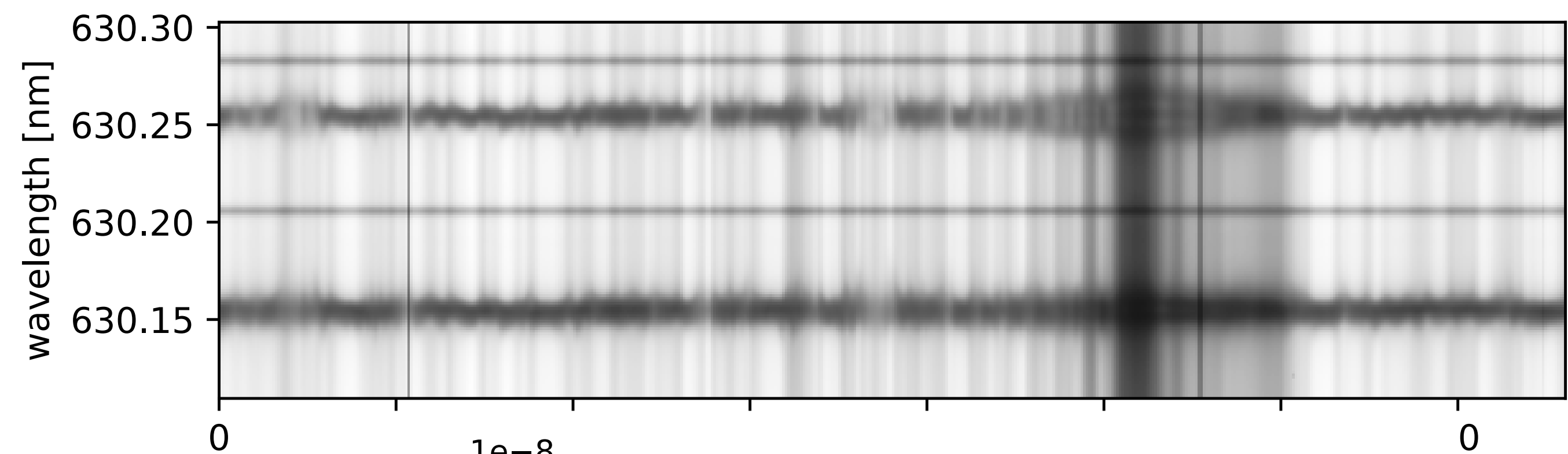
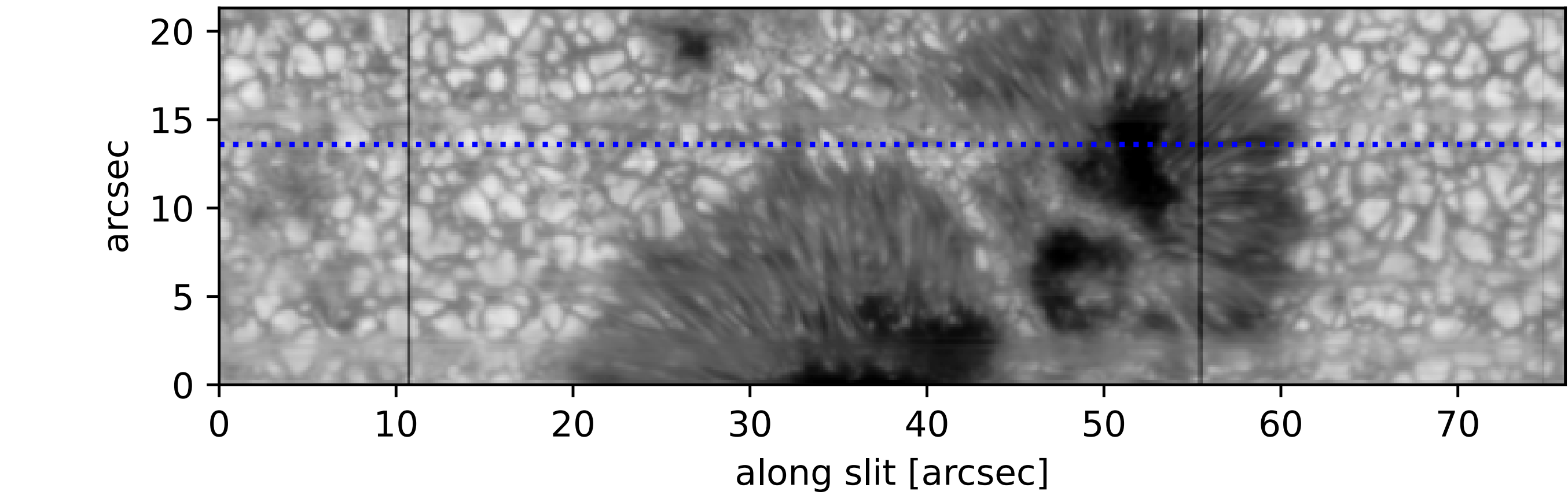
**Fig. 1** George Ellery Hale and his discovery of magnetic fields in sunspots through observations of the Zeeman effect. At the place where the spectrograph slit crosses a sunspot the spectral lines get split in polarized components. The magnitude of the splitting is proportional to the magnetic field strength, while the polarization state (circular or linear) reveals the orientation of the field.

# The Zeeman effect

DKIST/VBI G-band at 430.5 nm

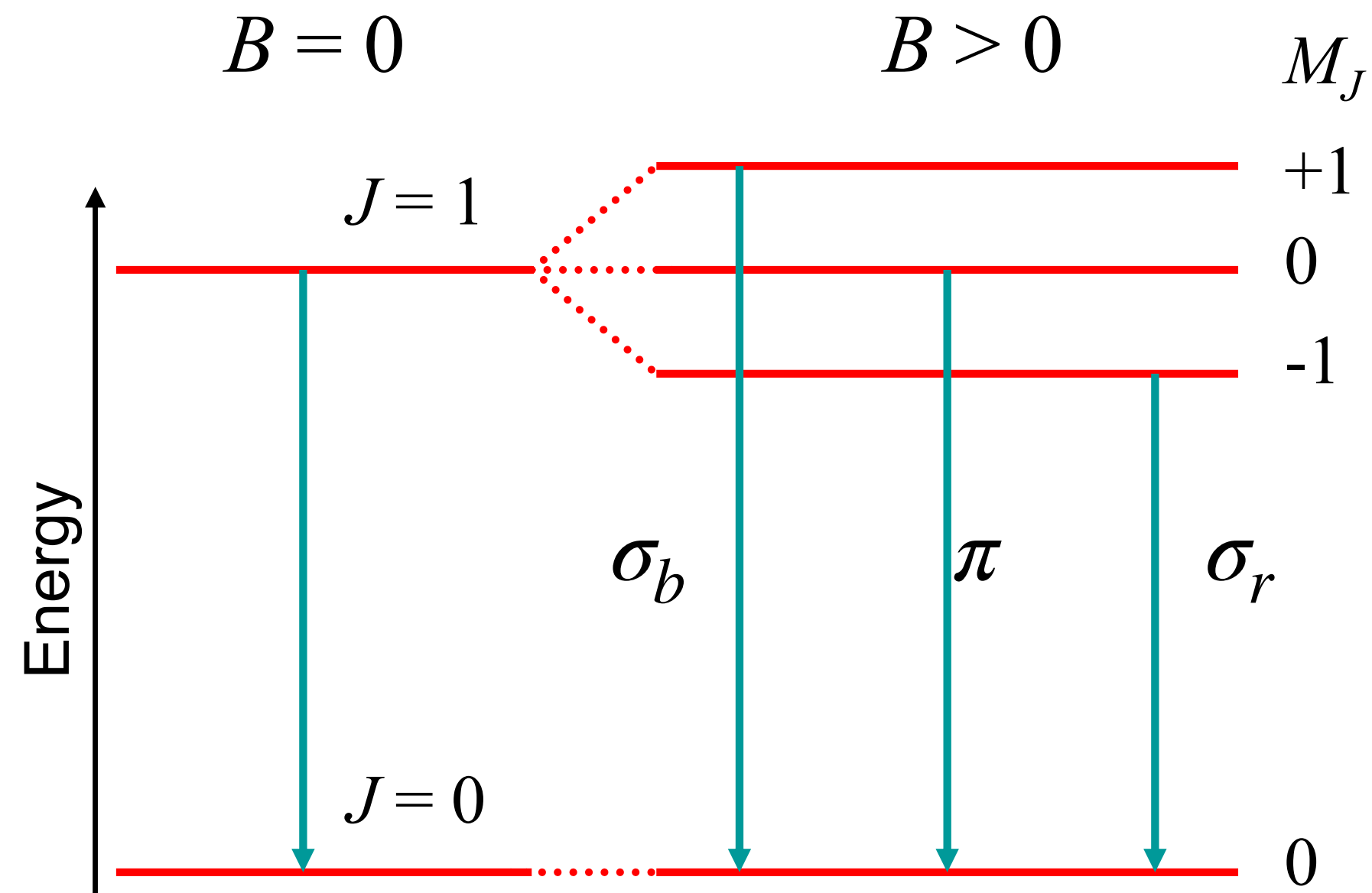


FeI 630.2 nm



$$\Delta\lambda_H = 4.67 \times 10^{-13} g_{eff} \lambda_0^2 B_{LOS}$$

# The Zeeman effect



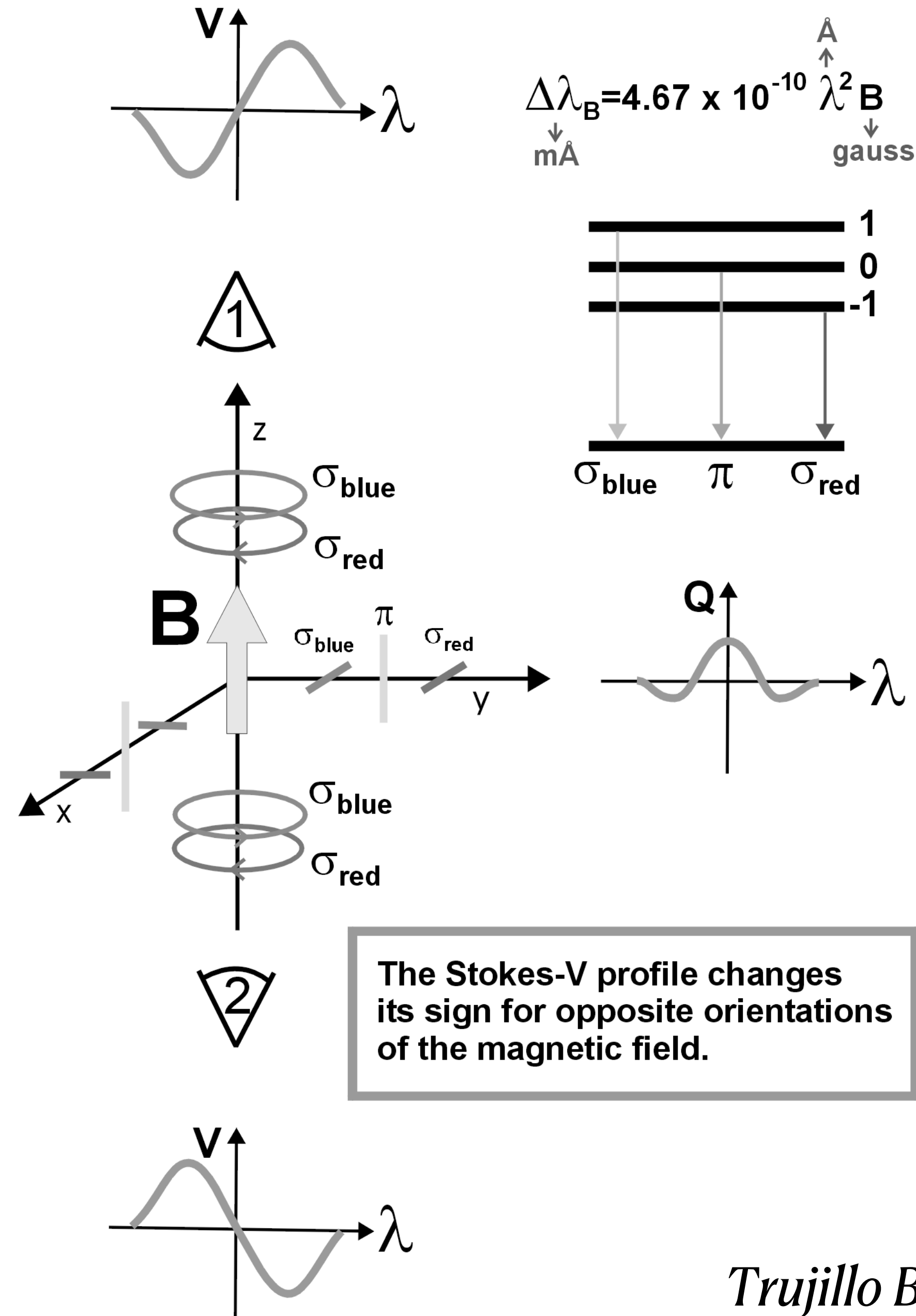
$$\Delta M = +1(\sigma_b), 0(\pi), -1(\sigma_r)$$

$$I = \odot + \ominus \text{ or } \leftrightarrow + \updownarrow \text{ etc}$$

$$Q = \updownarrow - \leftrightarrow,$$

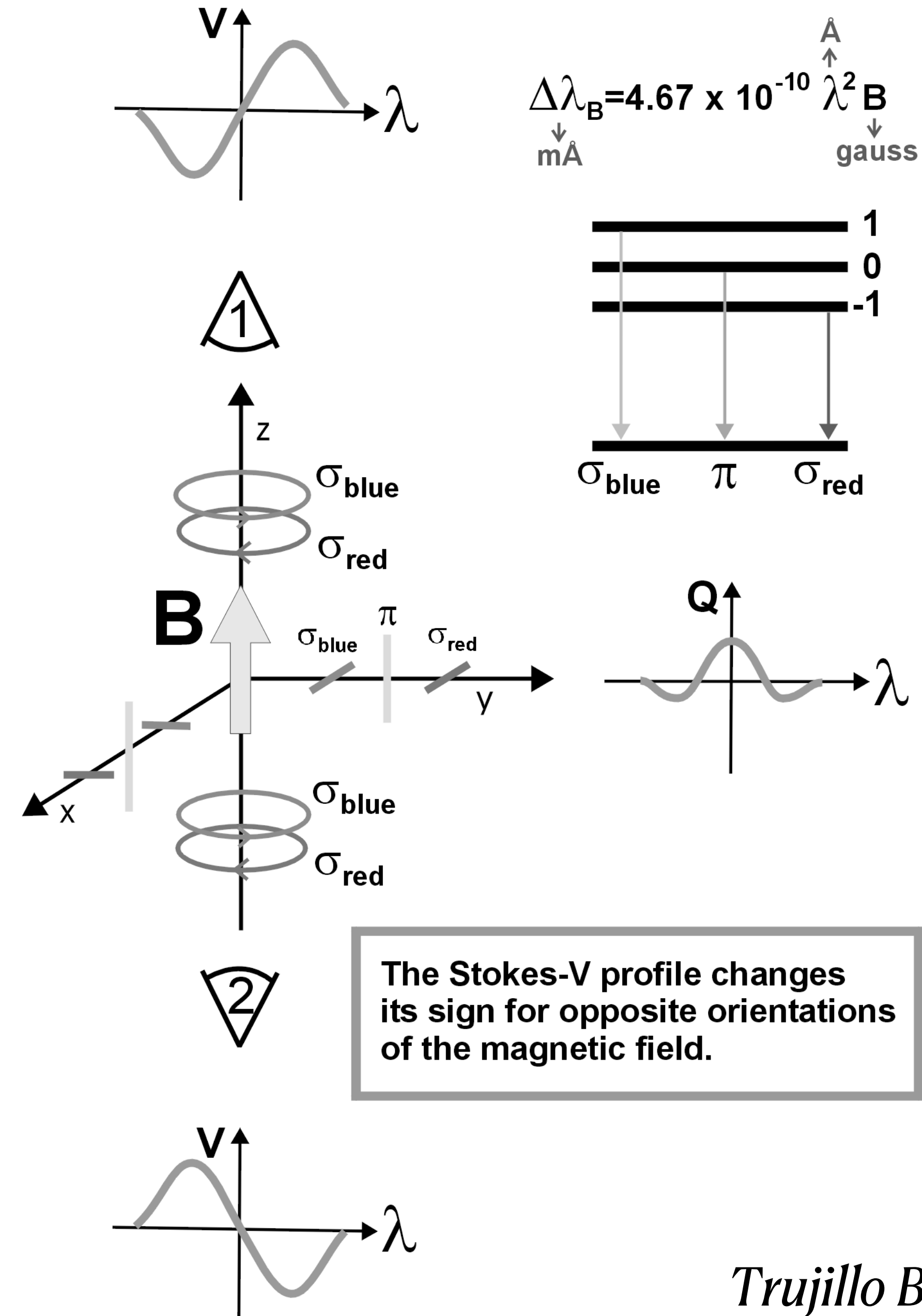
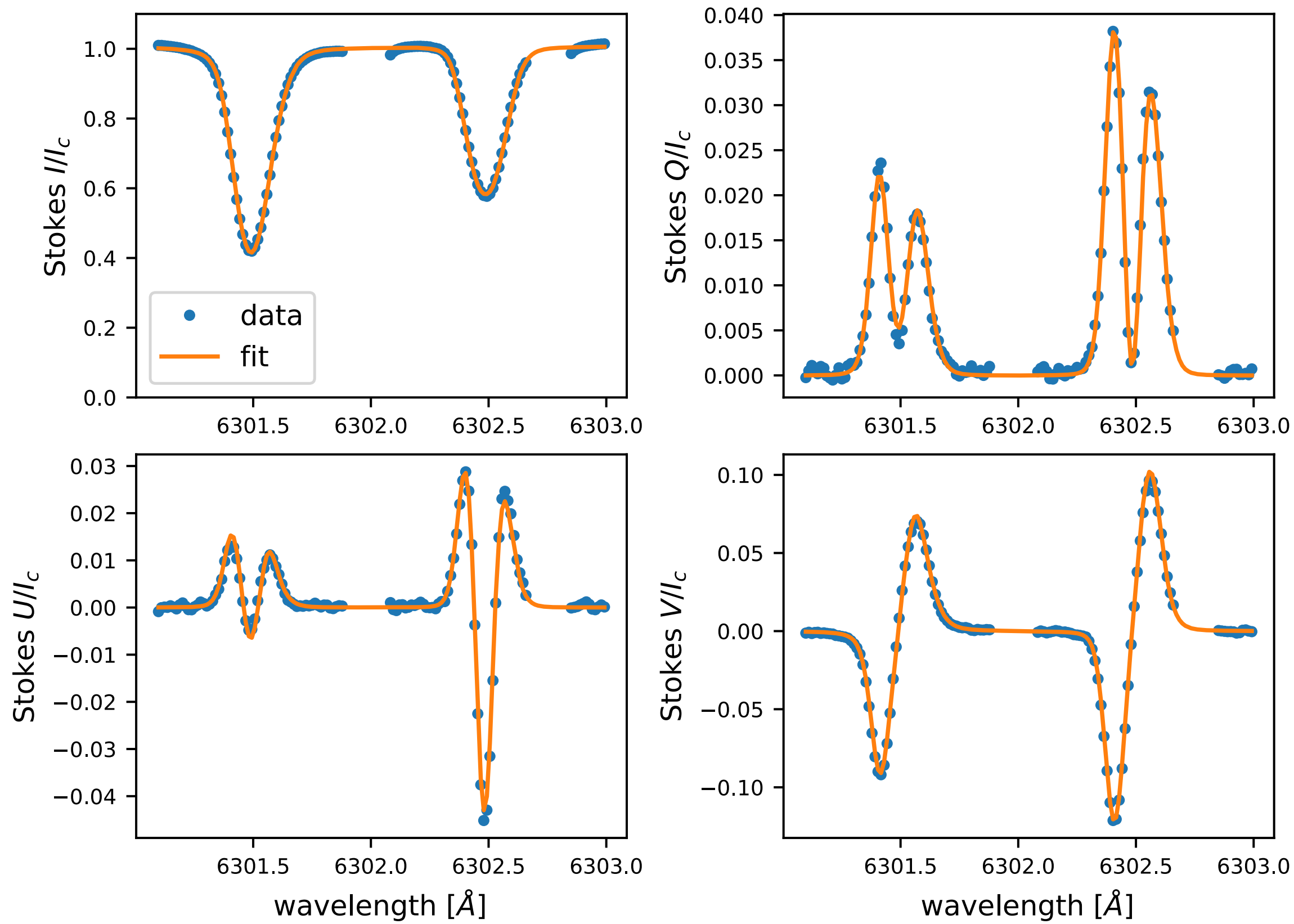
$$U = \swarrow - \searrow,$$

$$V = \odot - \ominus,$$



The Stokes-V profile changes its sign for opposite orientations of the magnetic field.

# The Zeeman effect





# Polarization observations with DKIST

## Measurements

- ViSP, DL-NIRSP, Cryo-NIRSP, VTF
- Modulation states are measured with rotating retarders/liquid crystals
- Dual beam polarimetry is used - analyzer is polarizing beam splitter
  - Minimizes cross-talks
  - Maximize S/N by using all available lights

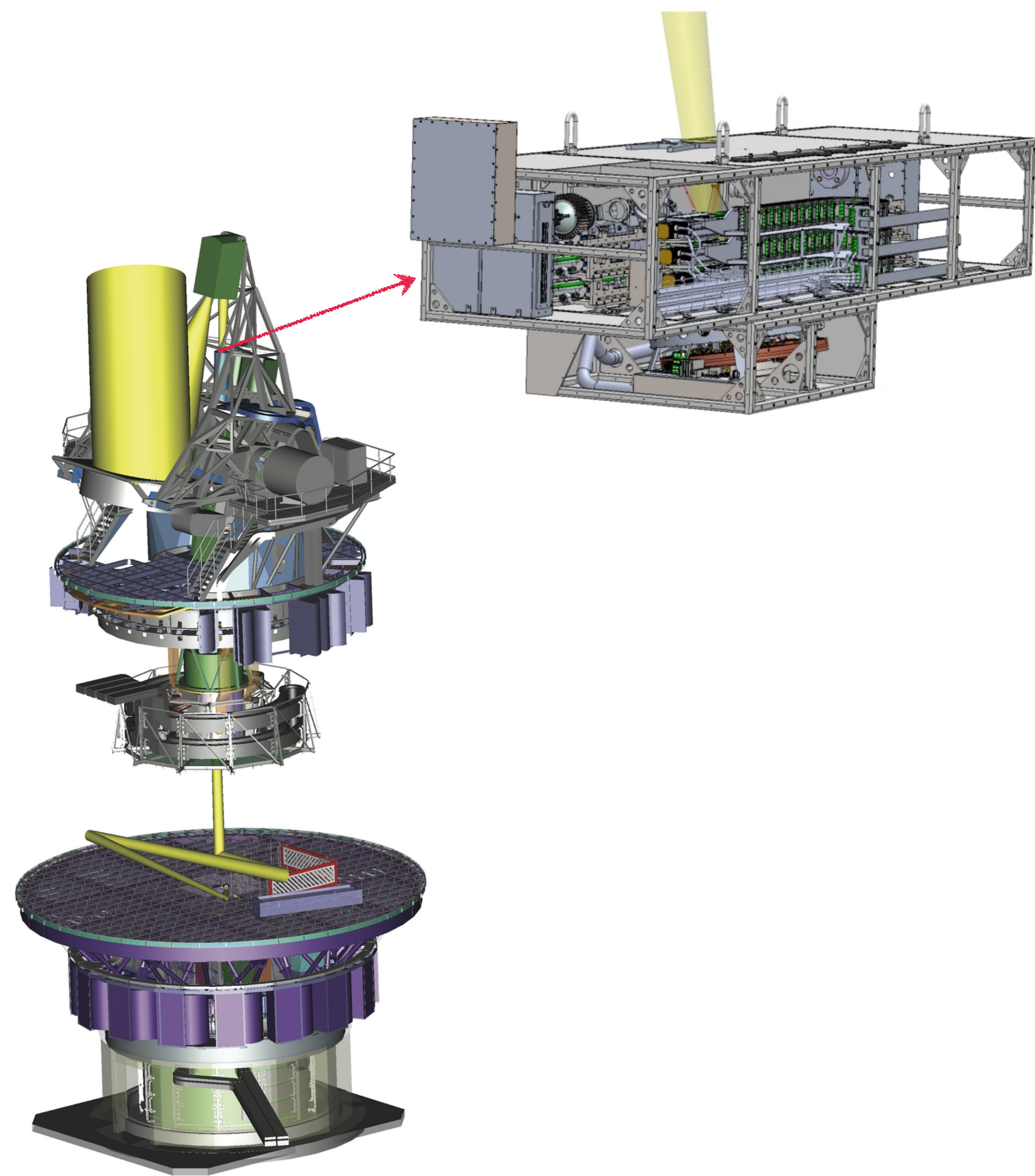
# Polarization observations with DKIST

## Calibration

- Combines two beams
- Convert modulation states into Stokes vector
- Removes polarizations induced by the telescope optics/instruments

# Polarization observations with DKIST

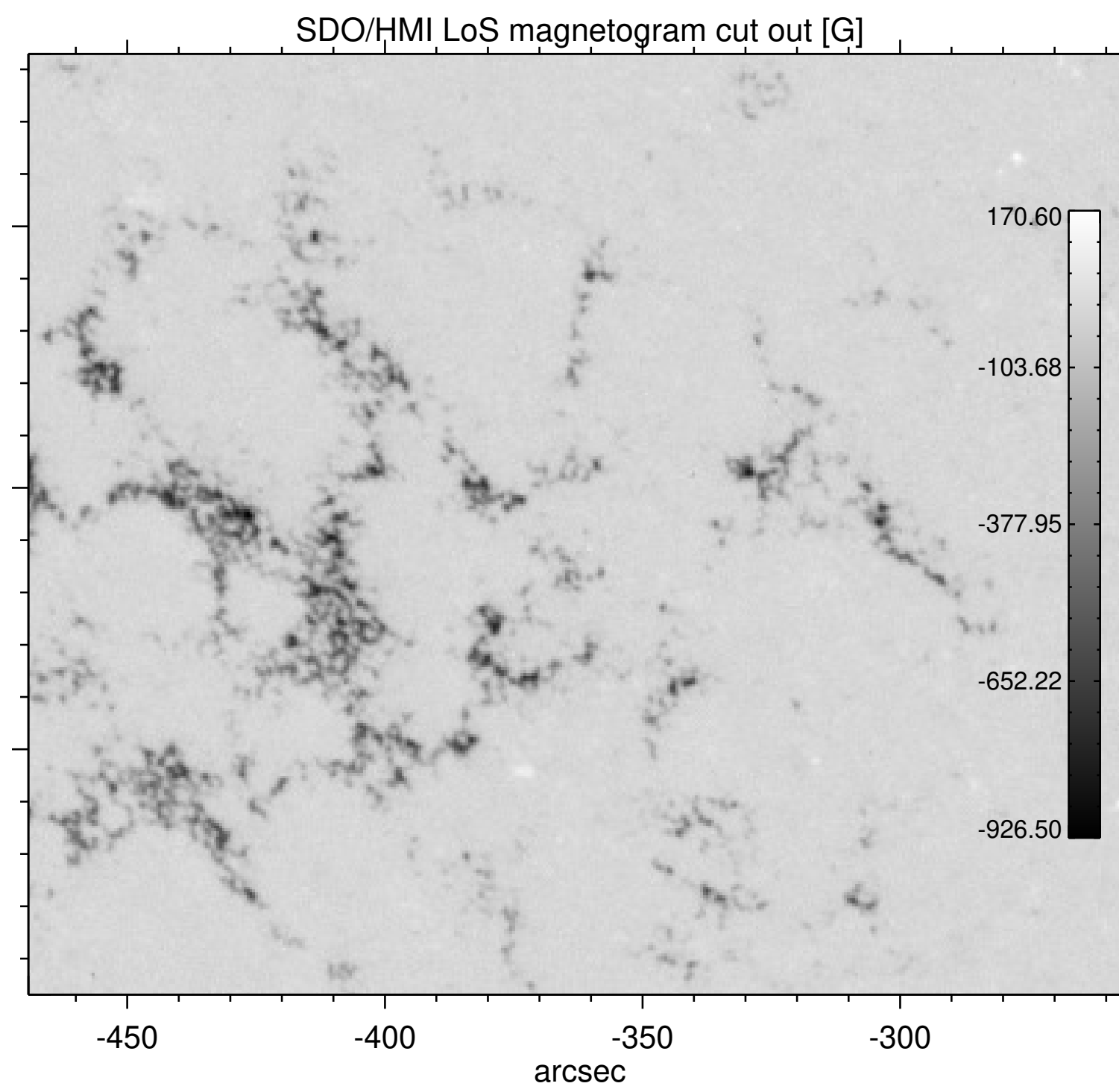
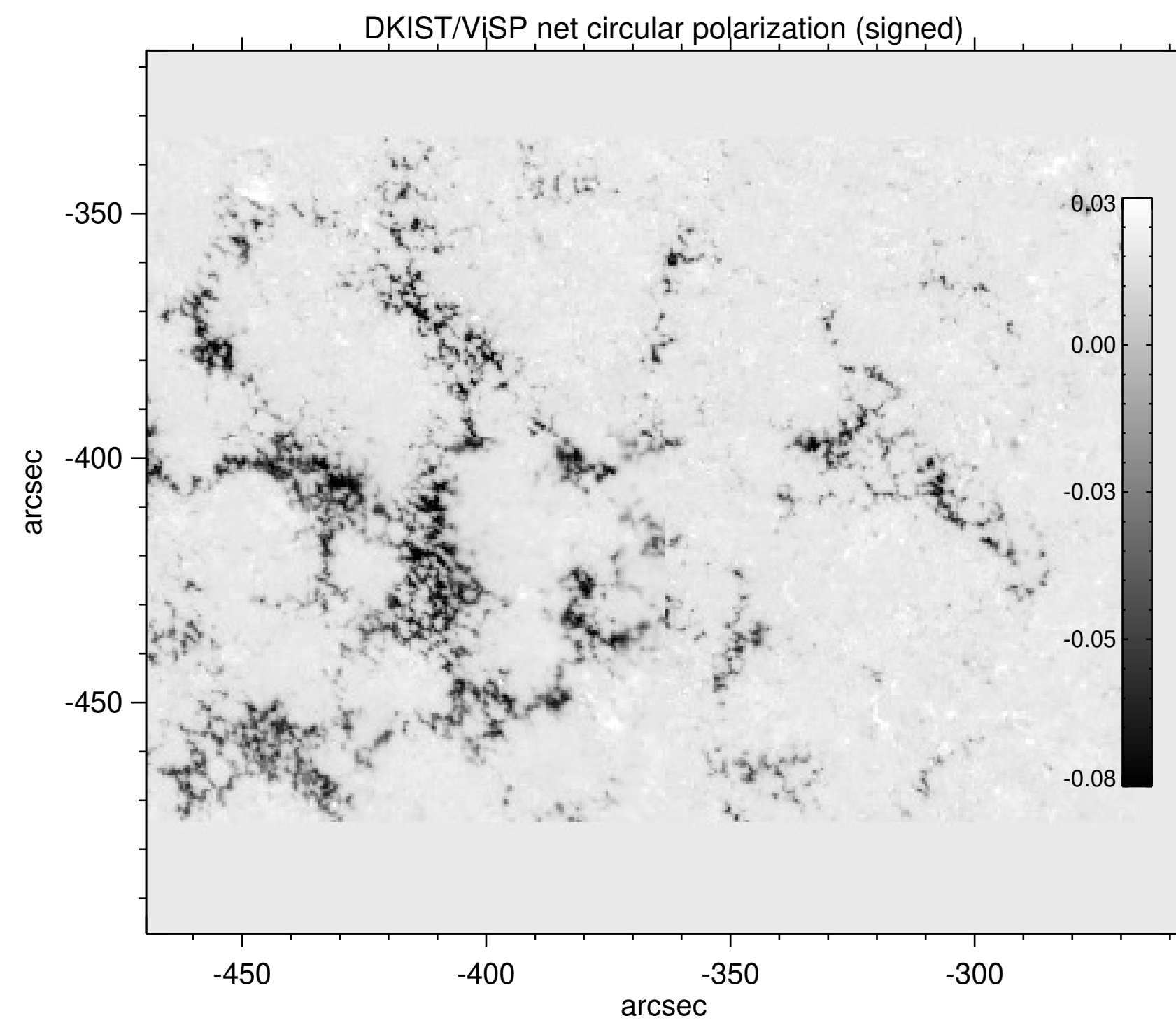
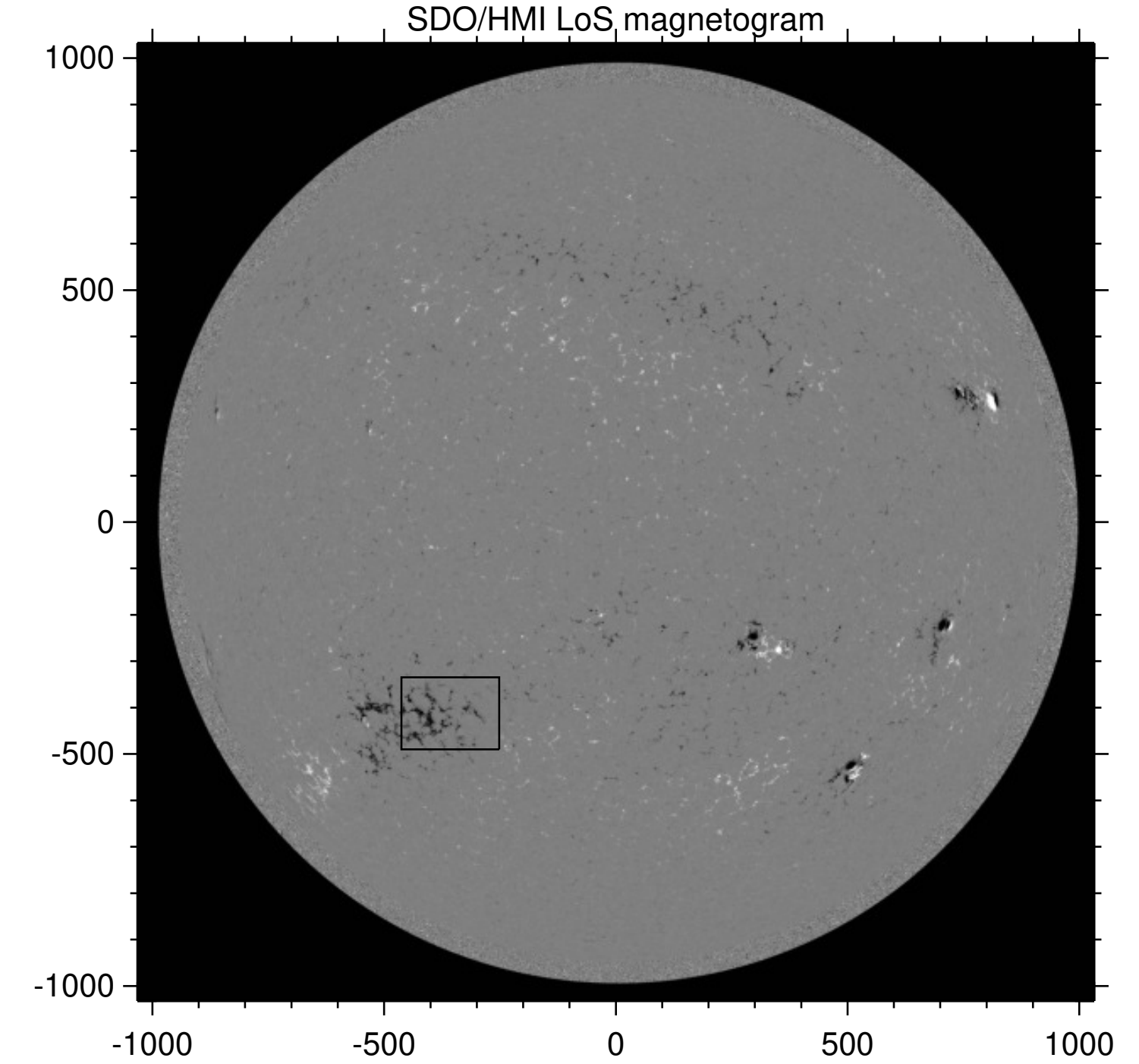
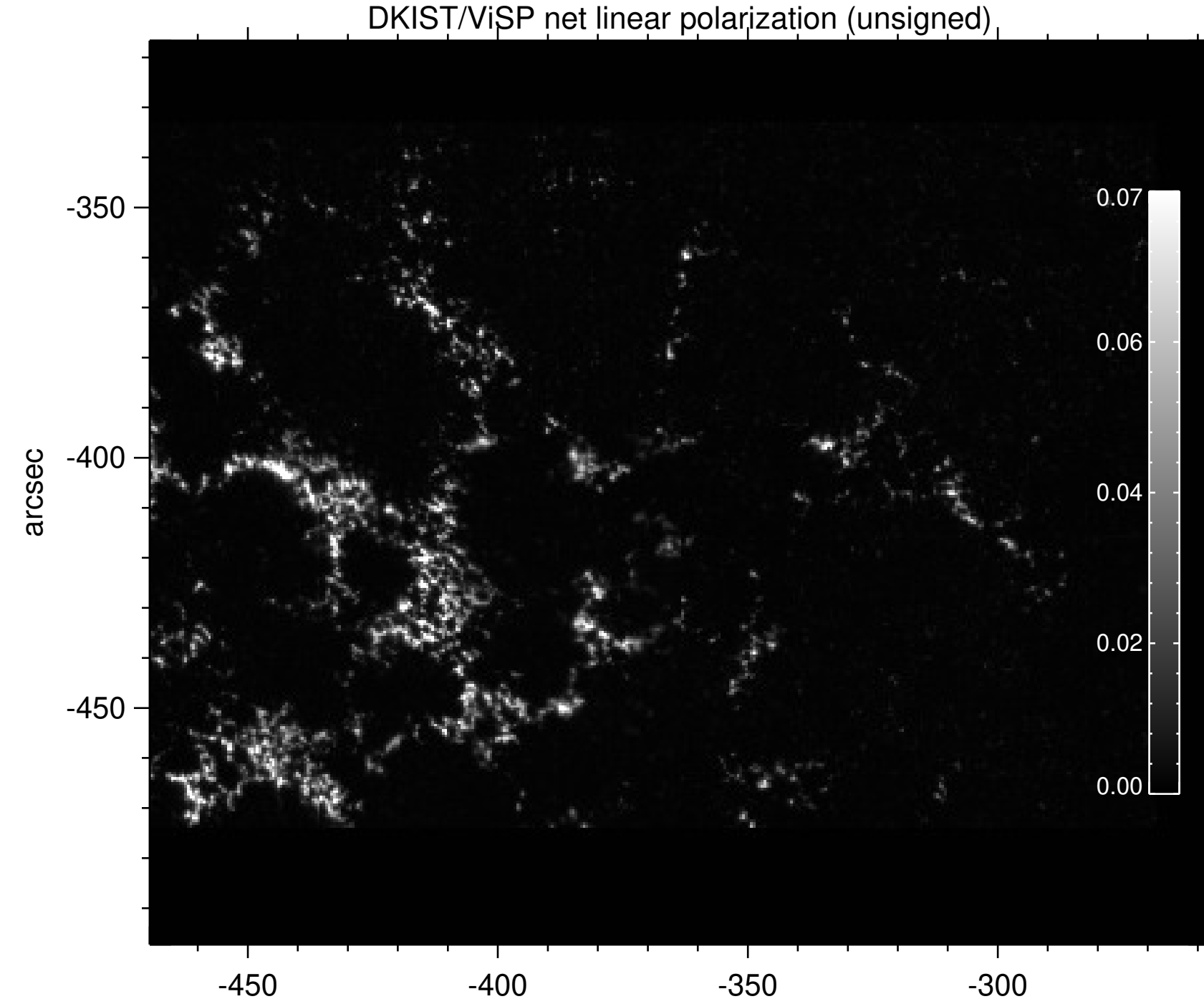
- DKIST use polarimetric calibration unit
- Placed near primary focus just after secondary mirror
- Has different optical elements including polarizers and retarders
- Modulation states are measured and analyzed in Coude lab level
- Polarization model (Muller matrix, telescope model etc)



# ViSP: Fe I 6301/6302

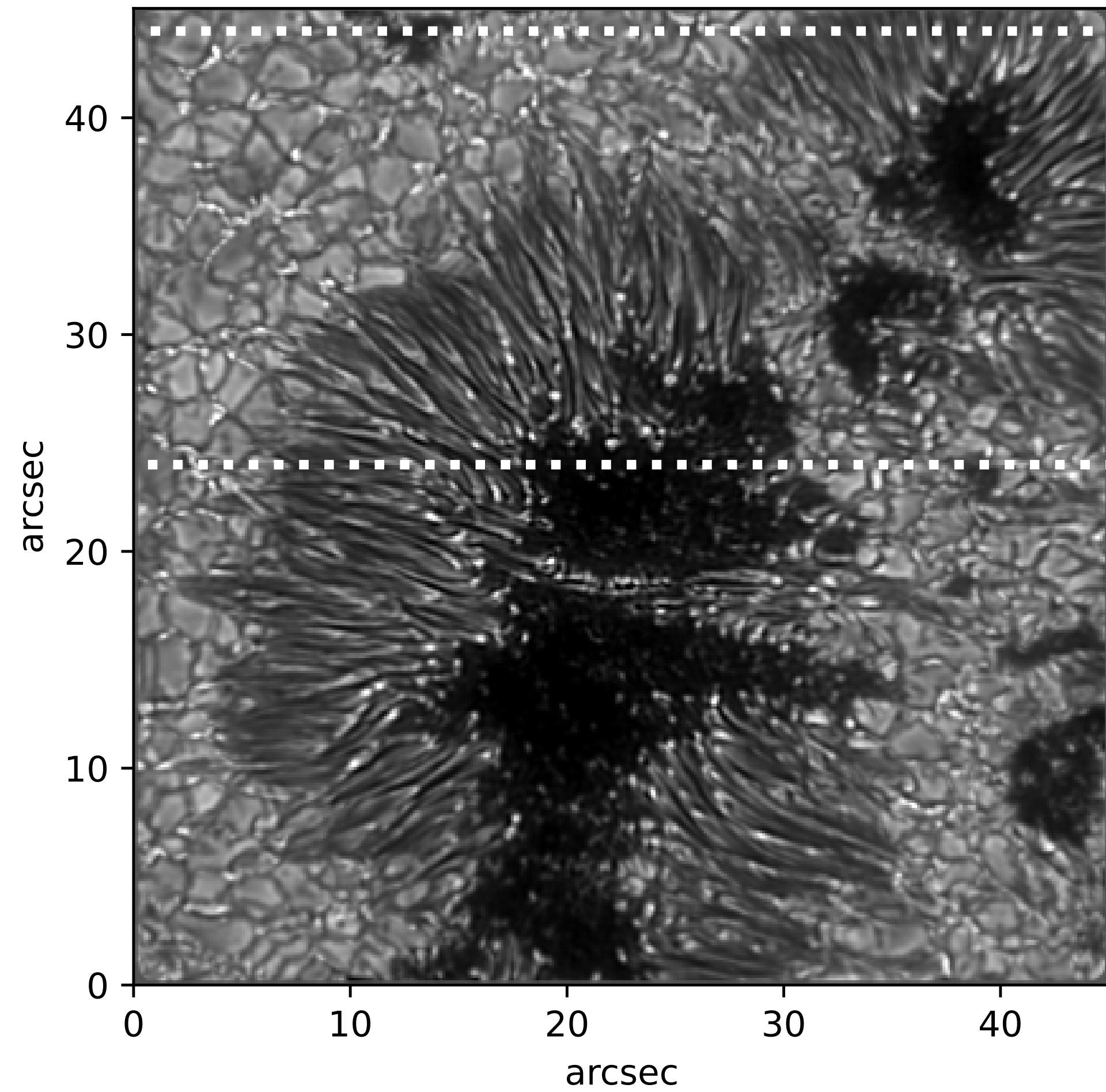
- **ViSP:** Fe I 6301/6302 Å, and Ca II 8542 Å
  - Slit width: 0.214"
  - Step size: 0.219" step
  - Full map time: ~27 min
  - Target: Plage region near  $\mu \sim 0.8$
  - 4 raster scan at 4 adjacent pointings

The noise level of the averaged Stokes  $Q$ ,  $U$ , &  $V$  profiles:  $4.4 \times 10^{-4} I_c$  and  $3.7 \times 10^{-4} I_c$

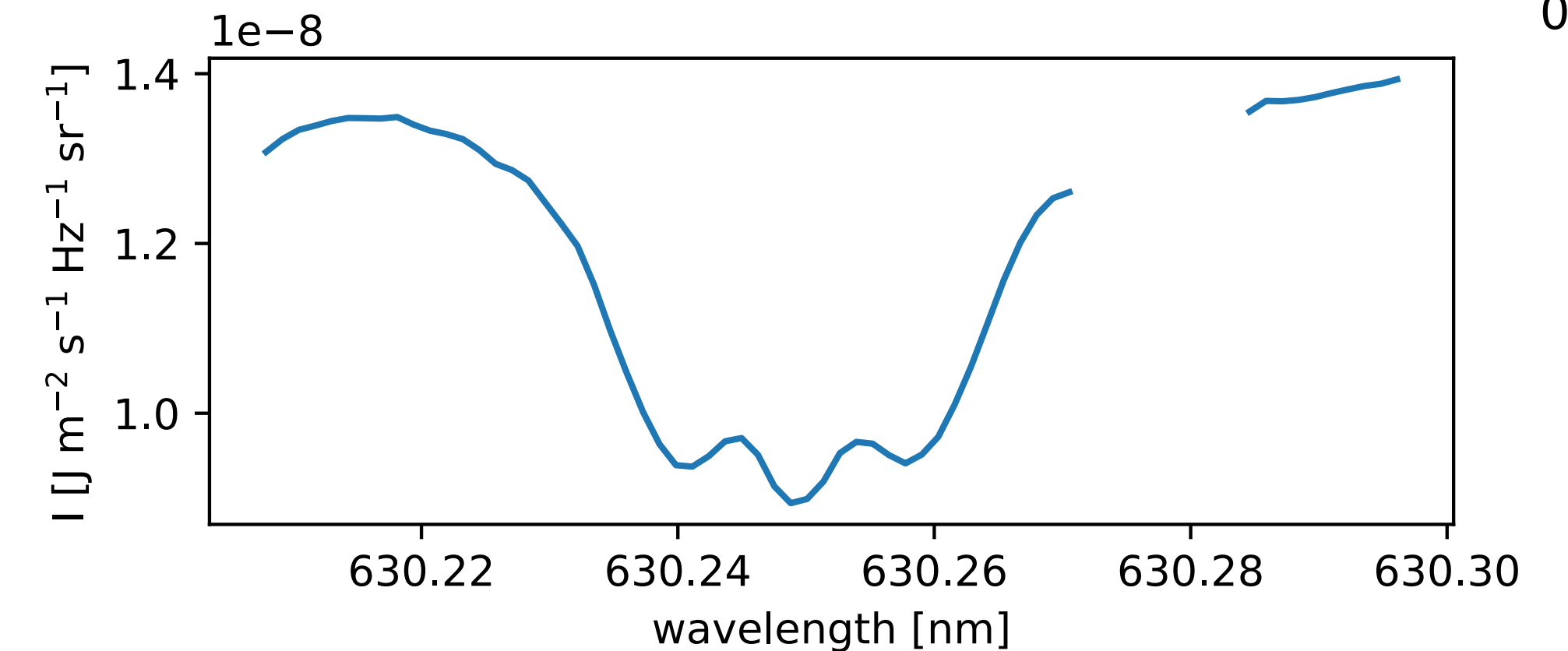
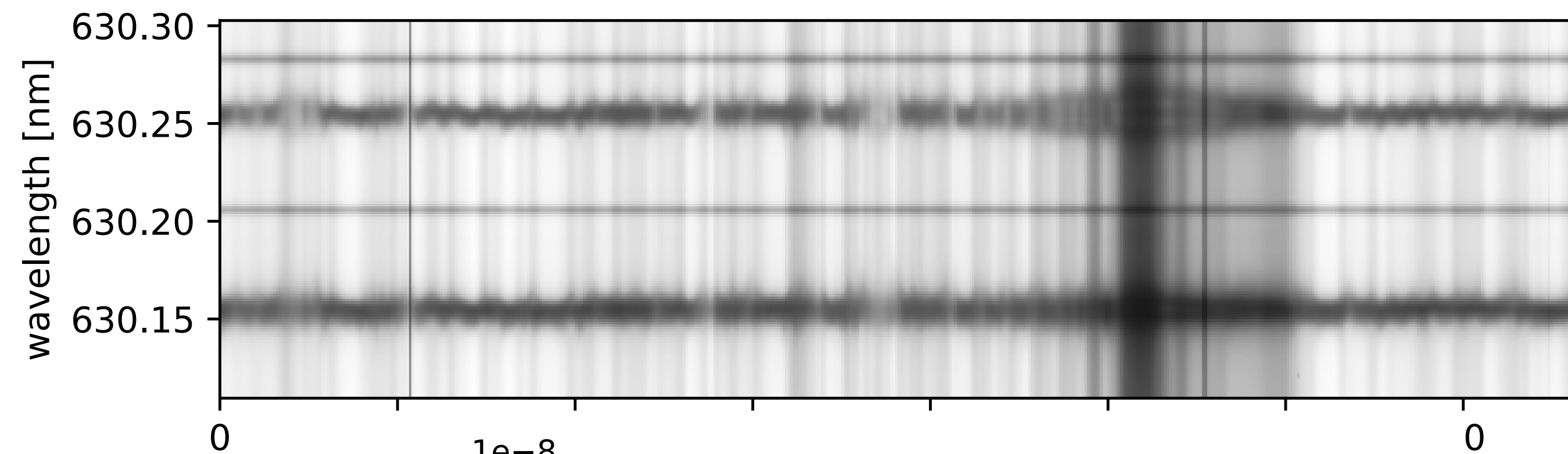
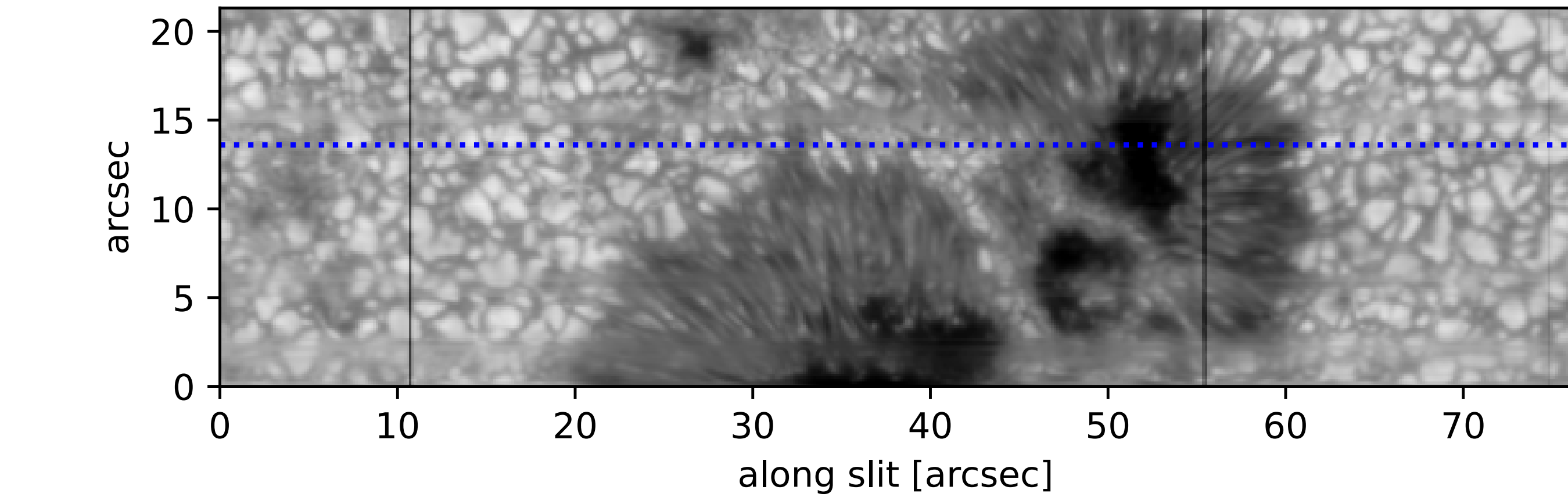


# The Zeeman effect - Strong Field Approximation

DKIST/VBI G-band at 430.5 nm



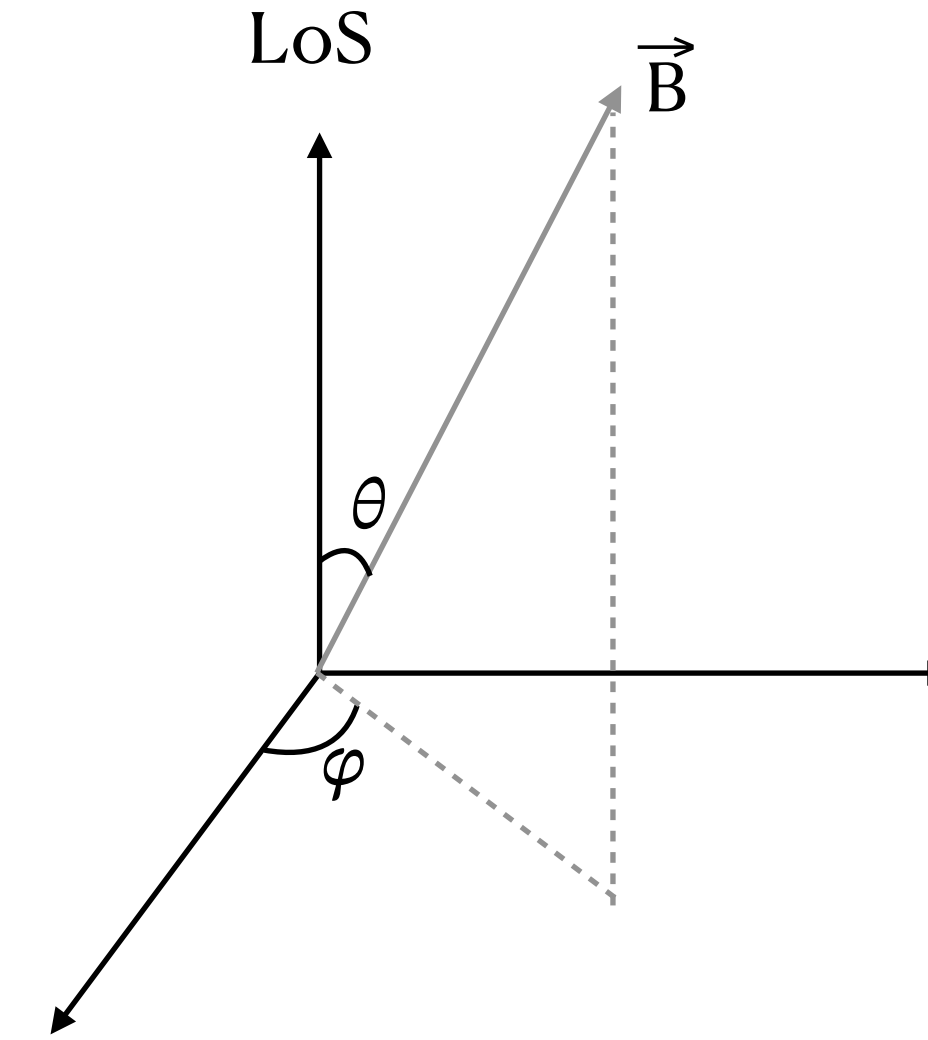
Fel 630.2 nm



$$\Delta\lambda_H = 4.67 \times 10^{-13} g_{eff} \lambda_0^2 B_{LOS}$$

# Polarized Radiative Transfer

$$\frac{d}{d\tau} \begin{pmatrix} I \\ Q \\ U \\ V \end{pmatrix} = \begin{pmatrix} \eta_I & \eta_Q & \eta_U & \eta_V \\ \eta_Q & \eta_I & \rho_V & -\rho_U \\ \eta_U & -\rho_V & \eta_I & \rho_Q \\ \eta_V & \rho_U & -\rho_Q & \eta_I \end{pmatrix} \begin{pmatrix} I - S \\ Q \\ U \\ V \end{pmatrix}$$



- $\eta_I$  - absorption coefficient
- $\eta_{Q,U,V}$  - describe the coupling of the intensity  $I$  with  $Q$ ,  $U$  and  $V$
- $\rho_{Q,U,V}$  - conversion terms (dispersion coefficients) between  $Q$ ,  $U$  and  $V$  due to magneto-optical effects

# Polarized Radiative Transfer

$$\frac{d}{d\tau} \begin{pmatrix} I \\ Q \\ U \\ V \end{pmatrix} = \begin{pmatrix} \eta_I & \eta_Q & \eta_U & \eta_V \\ \eta_Q & \eta_I & \rho_V & -\rho_U \\ \eta_U & -\rho_V & \eta_I & \rho_Q \\ \eta_V & \rho_U & -\rho_Q & \eta_I \end{pmatrix} \begin{pmatrix} I - S \\ Q \\ U \\ V \end{pmatrix}$$

$$\eta_I = 1 + \frac{\eta_0}{2} \left\{ \phi_0 \sin^2 \theta + \frac{1}{2} [\phi_{+1} + \phi_{-1}] (1 + \cos^2 \theta) \right\},$$

$$\eta_Q = \frac{\eta_0}{2} \left\{ \phi_0 - \frac{1}{2} [\phi_{+1} + \phi_{-1}] \right\} \sin^2 \theta \cos 2\varphi,$$

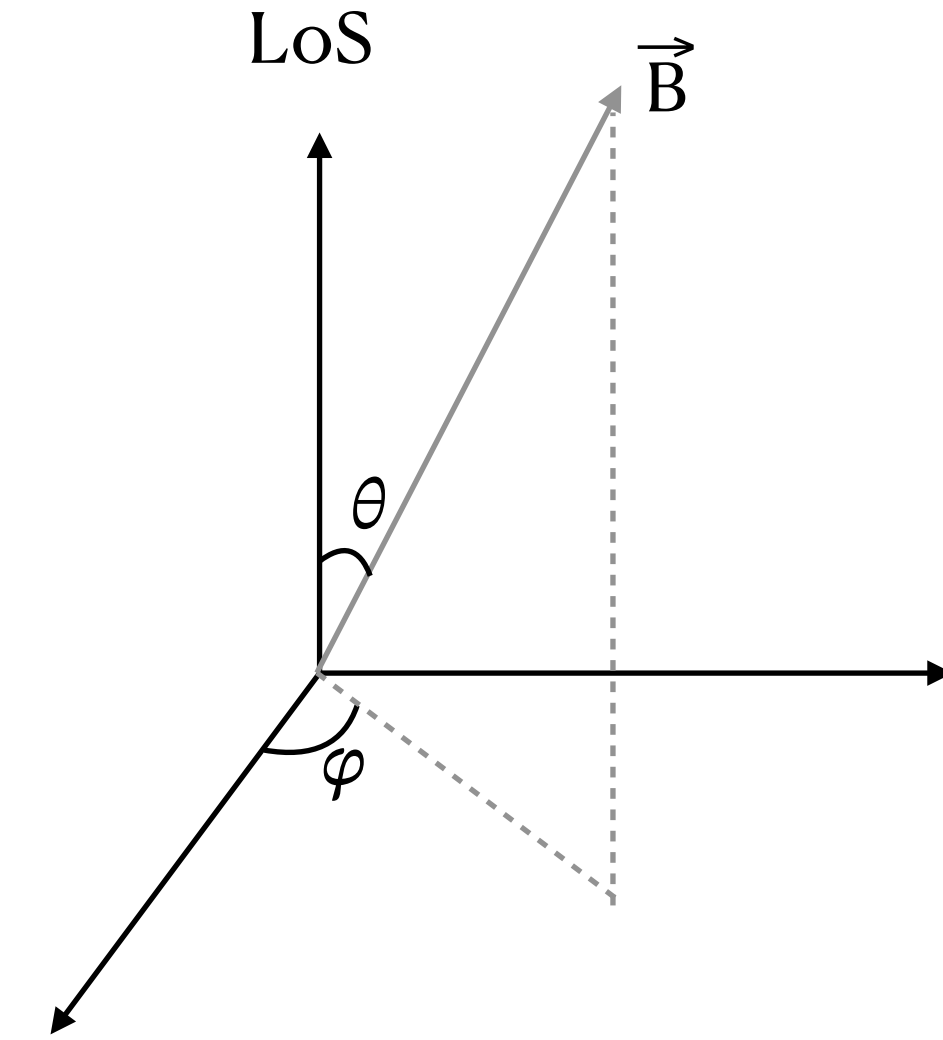
$$\eta_U = \frac{\eta_0}{2} \left\{ \phi_0 - \frac{1}{2} [\phi_{+1} + \phi_{-1}] \right\} \sin^2 \theta \sin 2\varphi,$$

$$\eta_V = \frac{\eta_0}{2} [\phi_{-1} - \phi_{+1}] \cos \theta,$$

$$\rho_Q = \frac{\eta_0}{2} \left\{ \psi_0 - \frac{1}{2} [\psi_{+1} + \psi_{-1}] \right\} \sin^2 \theta \cos 2\varphi,$$

$$\rho_U = \frac{\eta_0}{2} \left\{ \psi_0 - \frac{1}{2} [\psi_{+1} + \psi_{-1}] \right\} \sin^2 \theta \sin 2\varphi,$$

$$\rho_V = \frac{\eta_0}{2} [\psi_{-1} - \psi_{+1}] \cos \theta.$$



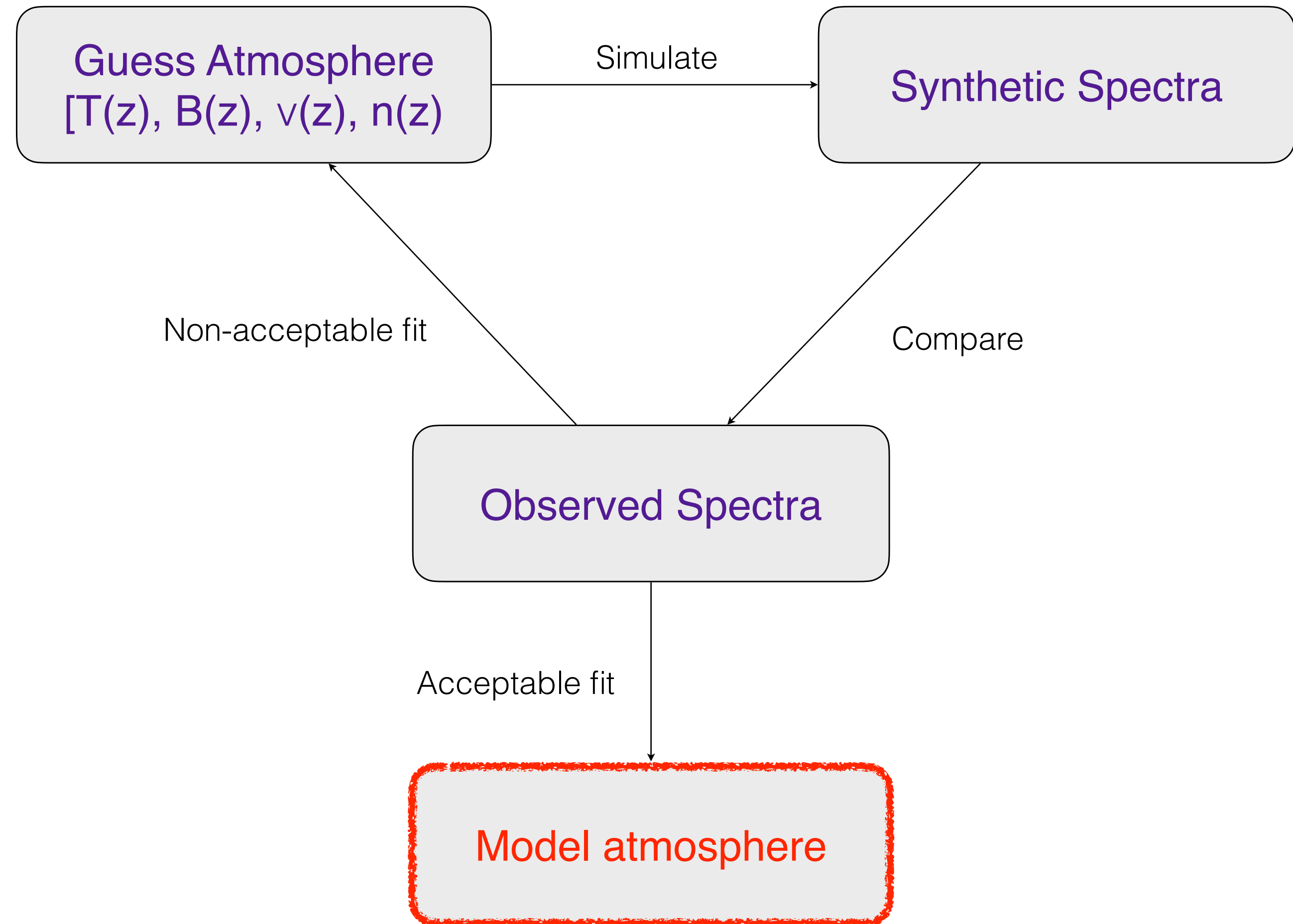
- $\theta$  &  $\varphi$  are the inclination and azimuth of the magnetic field
- $\phi_{0,+1,-1}$  components are computed from Voigt function that includes Doppler motion and Zeeman splitting
- Azimuthal angle  $\varphi$  appears as  $\sin(2\varphi)$  and  $\cos(2\varphi)$ 
  - The azimuth is determined by observations of Q,U only to within  $180^\circ$ .

# How we can infer magnetic field from Stokes profiles

- We need to solve a very complex radiative transfer problem under LTE/NLTE

## Inversions:

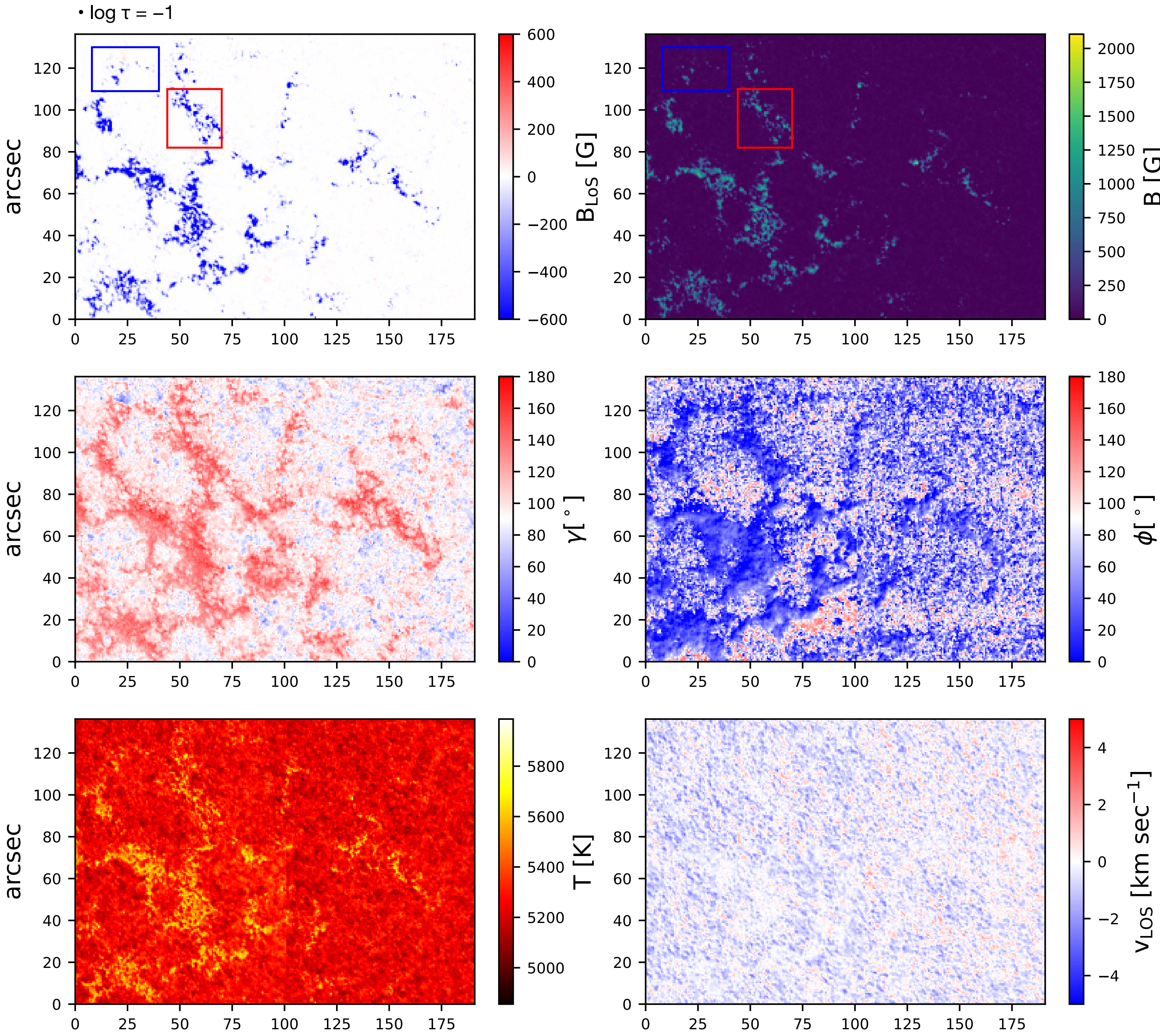
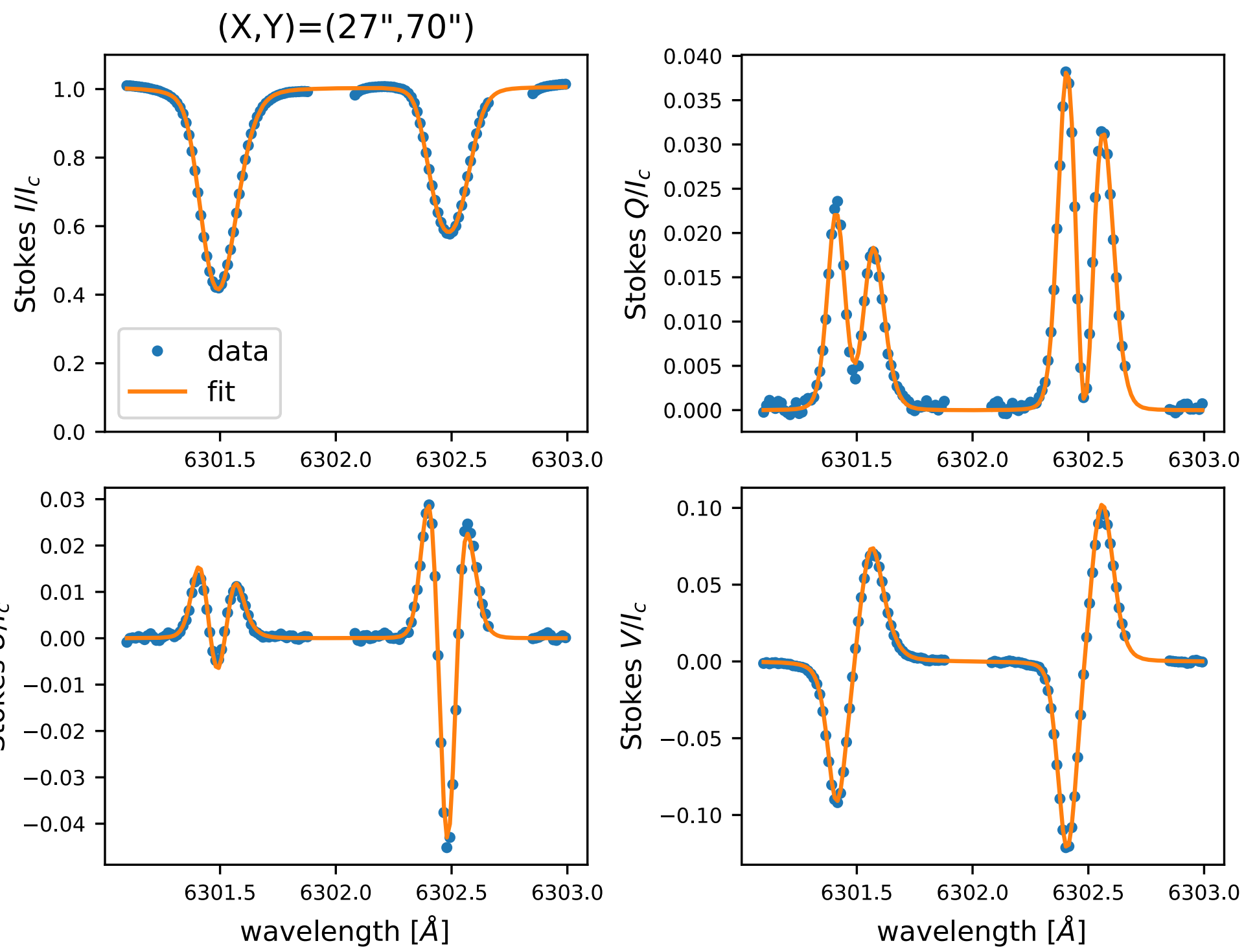
- Fit the observed Stokes profiles using inversion algorithms
- The ionization equilibrium, statistical equilibrium, and radiative transfer equations are solved numerically to synthesize the Stokes profiles



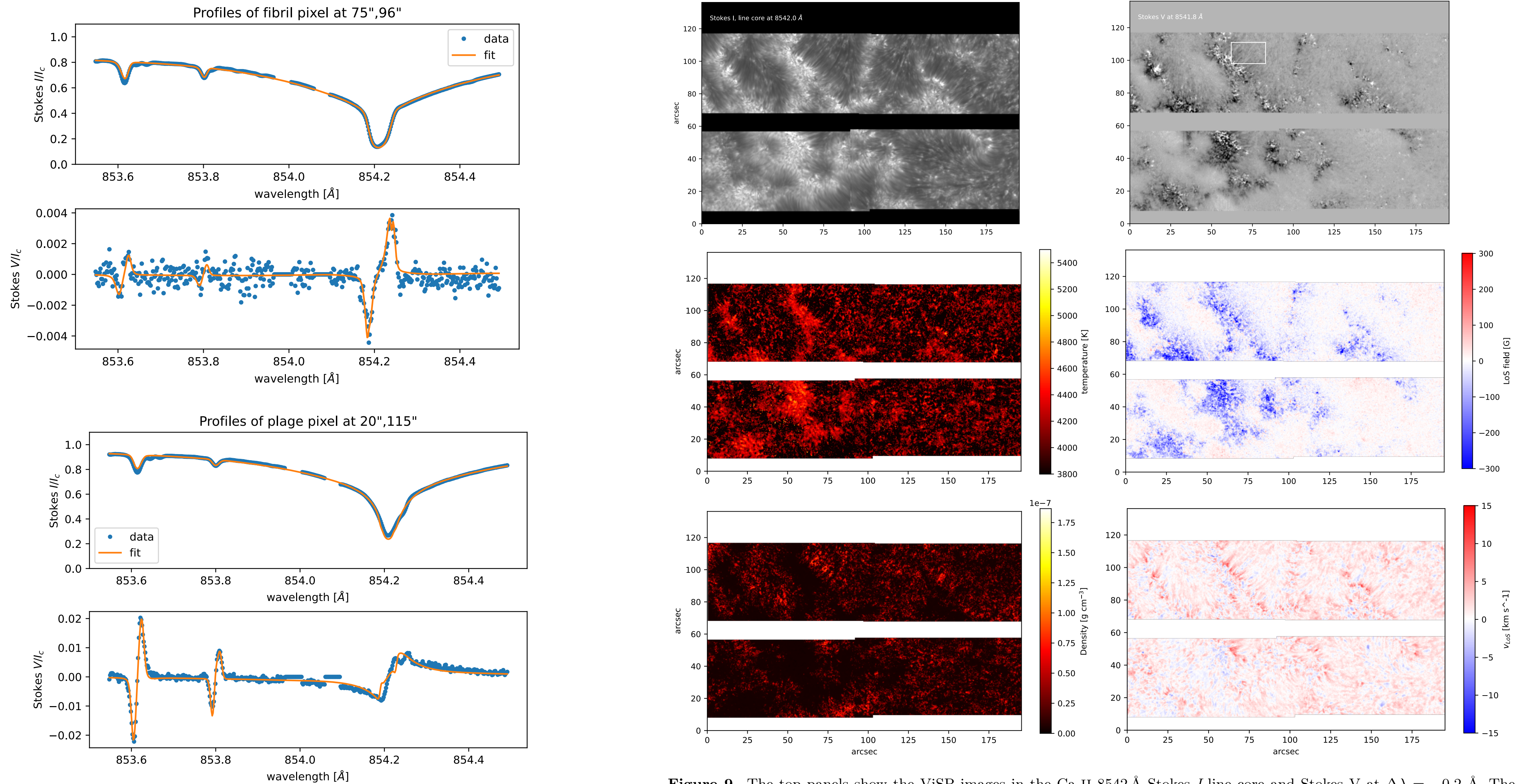


# Inversion of Fe 6301 & 6302 with DeSire code

- LTE inversion code SIR with non-LTE radiative transfer solver RH (Ruiz Cobo & del Toro Iniesta 1994, Uitenbroek 2001)
- The multilevel non-LTE radiative transfer problem using analytical response functions
- Plane-parallel geometry

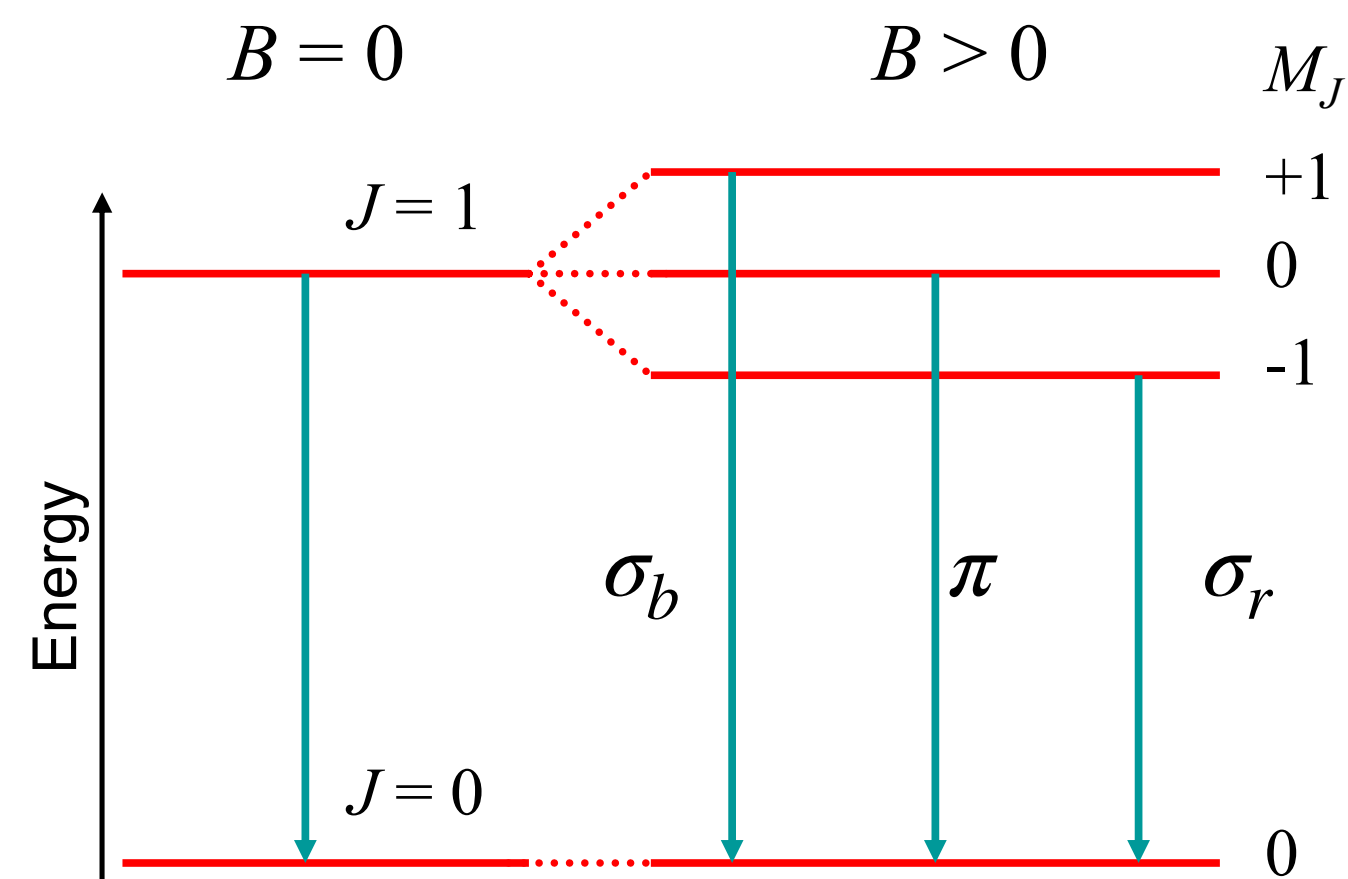


# Inversion of Ca II 8542 Å with DeSire code



**Figure 9.** The top panels show the ViSP images in the Ca II 8542 Å Stokes  $I$  line core and Stokes  $V$  at  $\Delta\lambda = -0.2$  Å. The DeSIRE output showing the temperature, LoS magnetic field, mass density and LoS velocity maps averaged over the interval between  $\log \tau = -2.8$  and  $-4.3$  are presented in the middle and bottom panels.

# Weak-field Approximation



$$V(\lambda) = \frac{1}{2} [I(\lambda - \Delta\lambda_H) - I(\lambda + \Delta\lambda_H)]$$

$$\Delta\lambda_H = 4.67 \times 10^{-13} g_{eff} \lambda_0^2 B_{LOS}$$

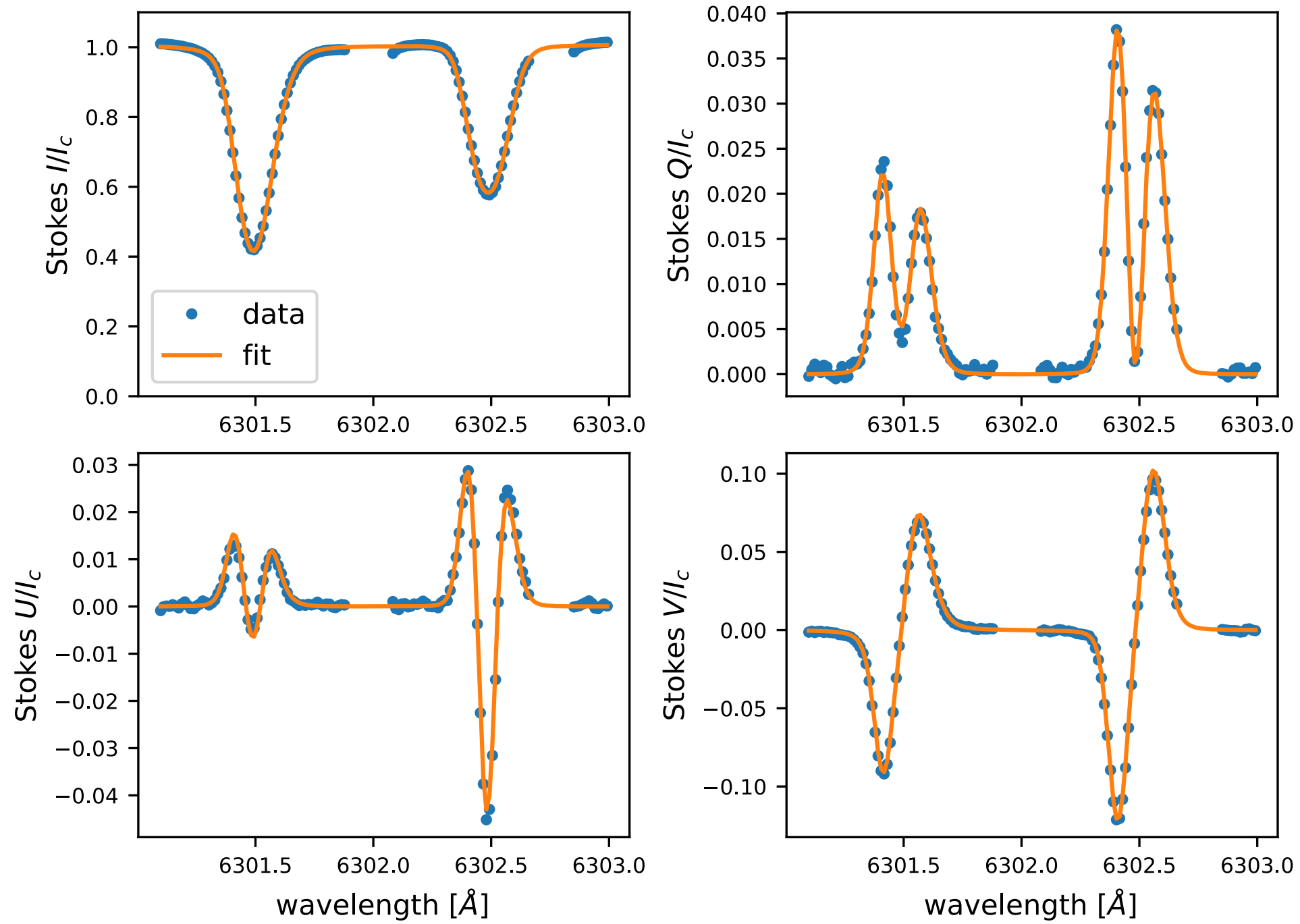
$$\Delta\lambda_D \gg \Delta\lambda_H \quad \Delta\nu_D = \frac{1}{\lambda_0} \sqrt{\frac{2kT}{m} + w_{nth}}$$

$$V = \Delta\lambda_H \left[ \frac{\partial I(\lambda)}{\partial \lambda} + \frac{1}{6} (\Delta\lambda_H^2) \frac{\partial^3 I(\lambda)}{\partial \lambda^3} + \dots \right]$$

$$V(\lambda) = -4.67 \times 10^{-13} g_{eff} \lambda_0^2 B_{LOS} \frac{\partial I(\lambda)}{\partial \lambda}$$

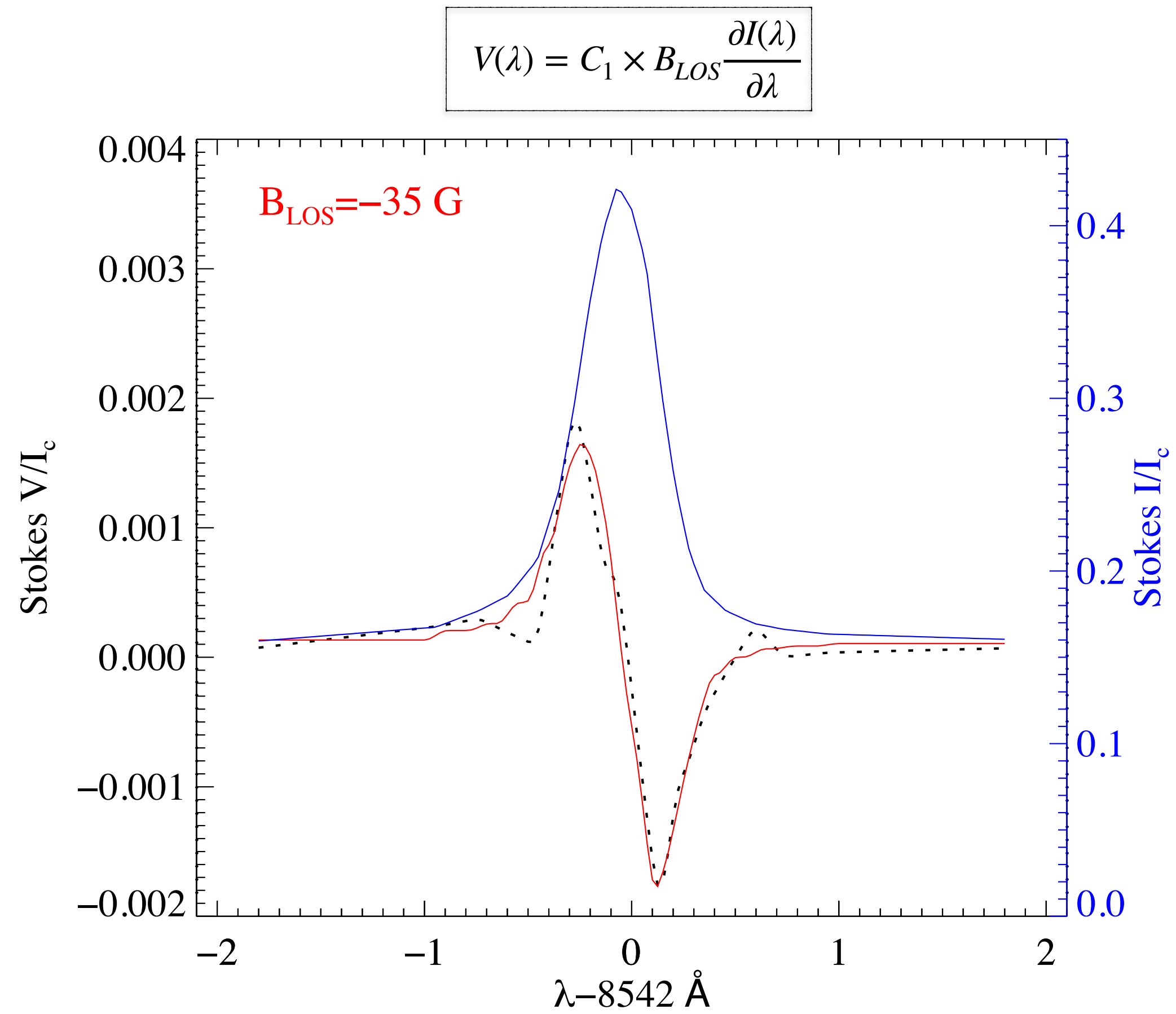
# Week-field Approximation

$$\Delta\lambda_D \gg \Delta\lambda_H$$

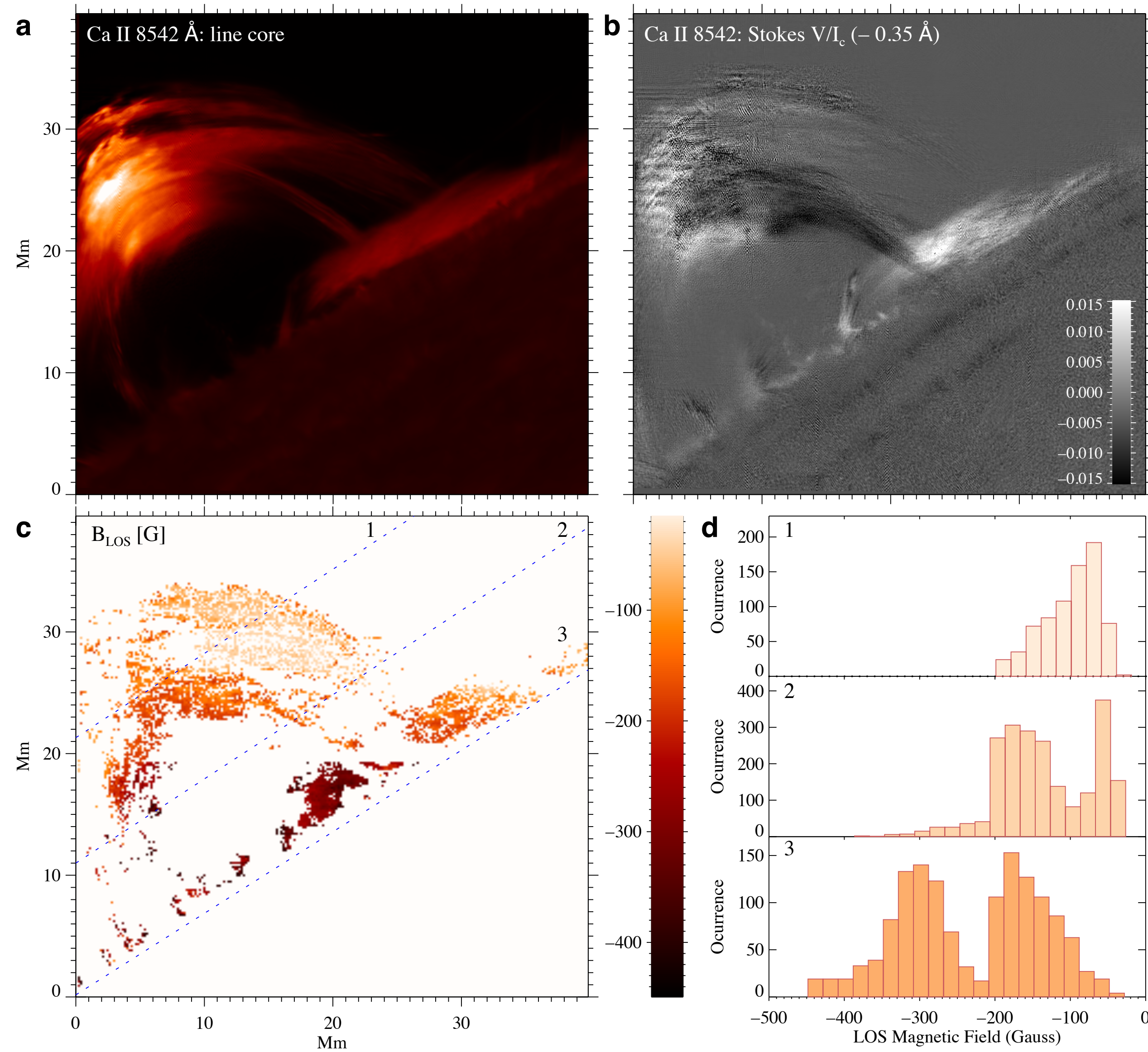


$$V(\lambda) = C_1 \times B_{LOS} \frac{\partial I(\lambda)}{\partial \lambda}$$
$$\sqrt{Q^2(\lambda) + U^2(\lambda)} = C_2 \times B_{\perp} \frac{\partial^2 I(\lambda)}{\partial \lambda^2}$$

# Week-field Approximation



# X8.2 flare coronal loops on 10 September 2017 observed by SST



Panel *a* and *b* show the SST images in the Ca ii 8542 Å intensity (Stokes *I*) at line core and circular polarization (Stokes *V*) at line wing for the flare coronal loops. Panel *c* and *d* show a map of the LOS magnetic field together with histogram showing the distribution of the LOS magnetic fields for 3 different regions (1, 2, 3).

# Center of Gravity (COG) method

The COG is defined as the centroid of its residual intensity profiles:

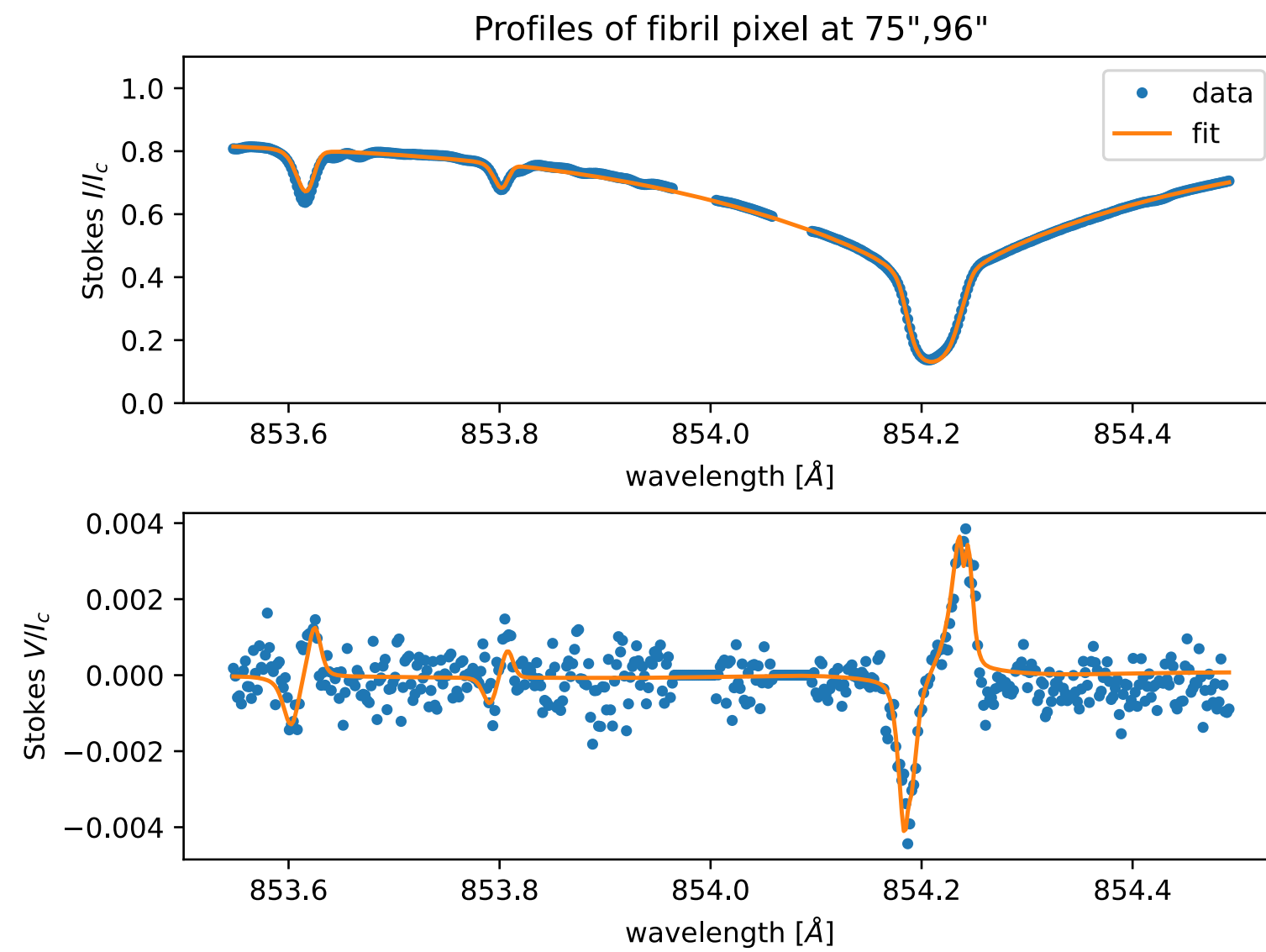
$$\lambda_{COG} = \frac{\int \lambda(I_{cont} - I)d\lambda}{\int (I_{cont} - I)d\lambda}$$

$$v_{LOS} = \frac{c(\lambda_0 - \lambda_{COG})}{\lambda_0}$$

The LOS strength of magnetic field

$$B_{LOS} = \frac{\lambda_+ - \lambda_-}{2} \frac{4\pi mc}{eg_L \lambda_0^2}$$

$$\lambda_{\pm} = \frac{\int \lambda(I_{cont} - (I \pm V))d\lambda}{\int (I_{cont} - (I \pm V))d\lambda}$$



# Summary

- The most powerful magnetic diagnostics of solar and astrophysical plasmas are based on measurements of the polarized states of light described by the Stokes profiles  $(I, Q, U, V)$
- Measurement of magnetic field in the photosphere and chromosphere is mainly based on Zeeman effect
- Simple techniques such as weak-field/strong-field approximation, center-of-gravity can be used to produce maps of the magnetic field components